Seasonal Arima

Lecture 11

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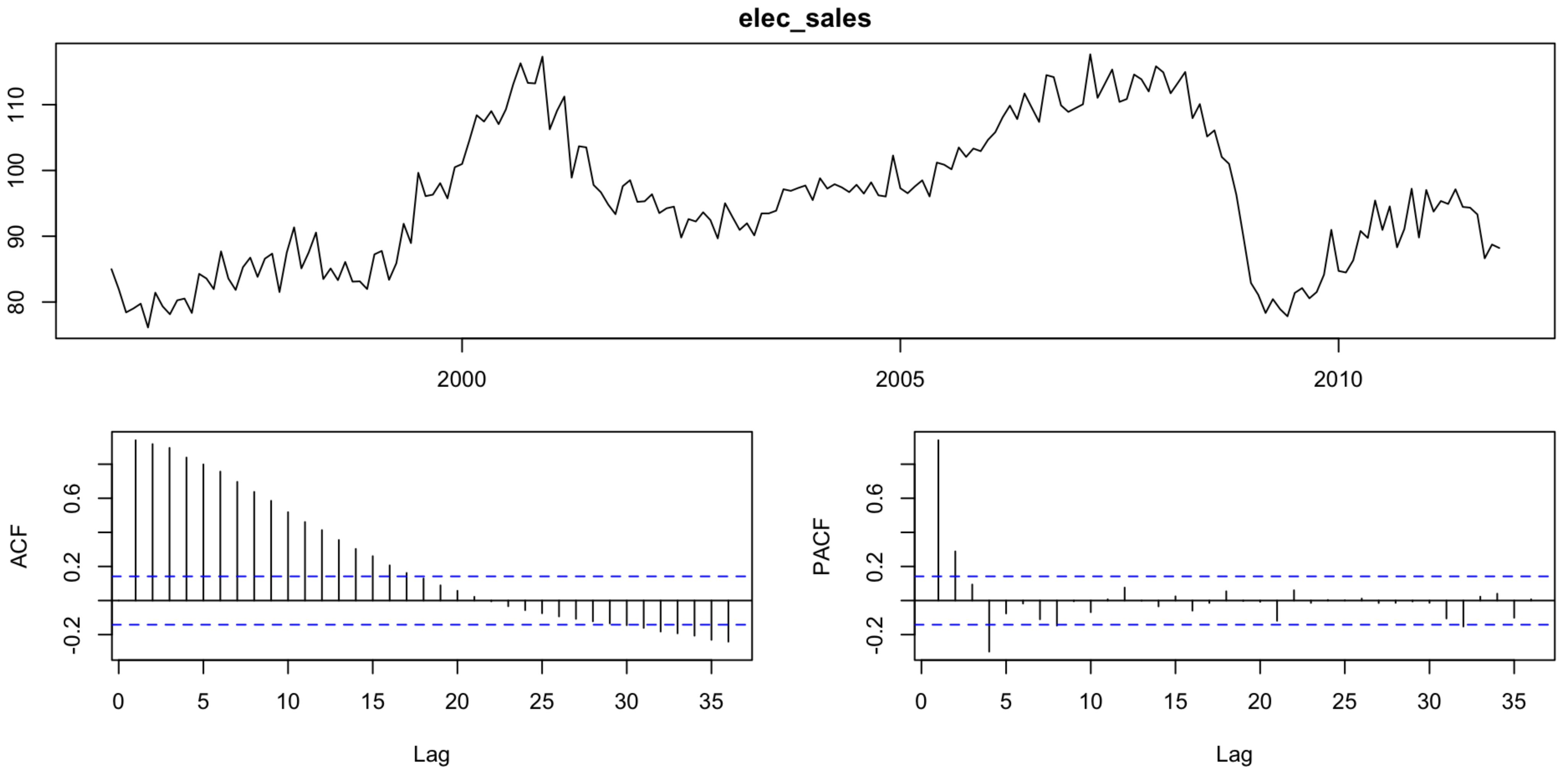
ARIMA - General Guidance

1. Positive autocorrelations out to a large number of lags usually indicates a need for differencing
2. Slightly too much or slightly too little differencing can be corrected by adding AR or MA terms respectively.
3. A model with no differencing usually includes a constant term, a model with two or more orders (rare) differencing usually does not include a constant term.
4. After differencing, if the PACF has a sharp cutoff then consider adding AR terms to the model.
5. After differencing, if the ACF has a sharp cutoff then consider adding an MA term to the model.
6. It is possible for an AR term and an MA term to cancel each other's effects, so try models with fewer AR terms and fewer MA terms.

Based on guidance in Robert Nau's notes on regression and time series analysis - Chapter 5

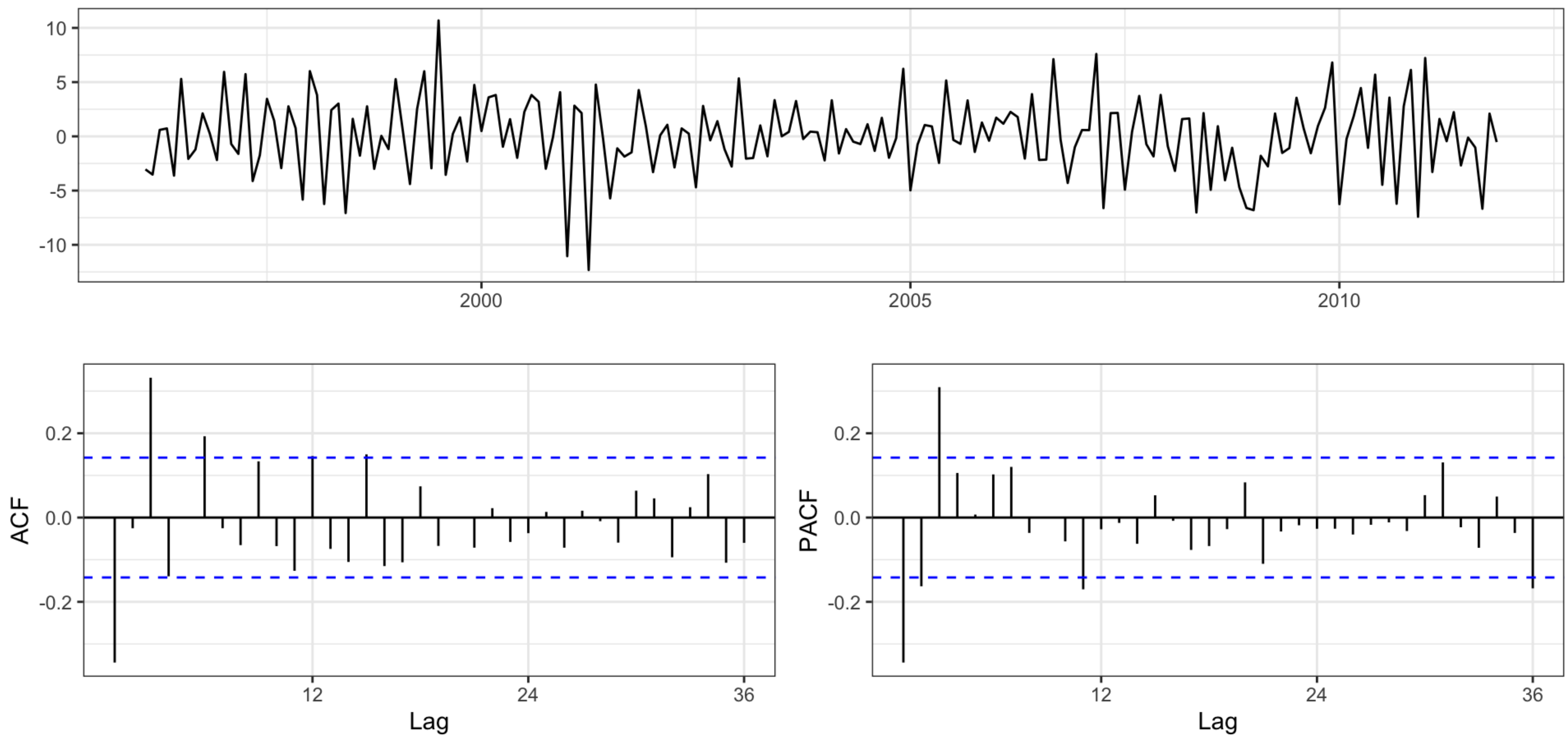
Electrical Equipment Sales

Data

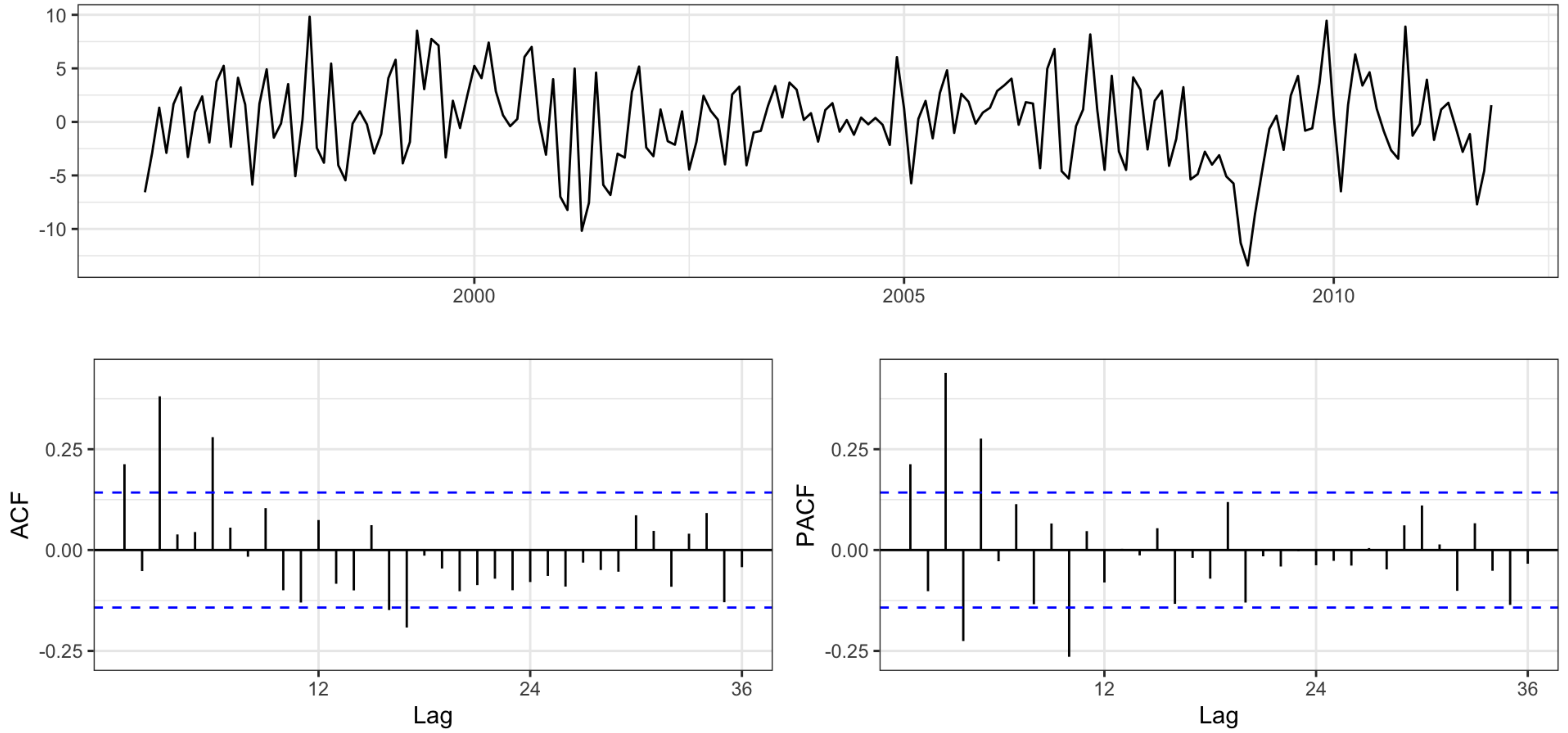


elec_sales
80
90
100
110
2000
2005
2010
ACF
-0.2
0.2
0.6
0
5
10
15
20
25
30
35
Lag
PACF
-0.2
0.2
0.6
0
5
10
15
20
25
30
35
Lag

1st order differencing



2nd order differencing



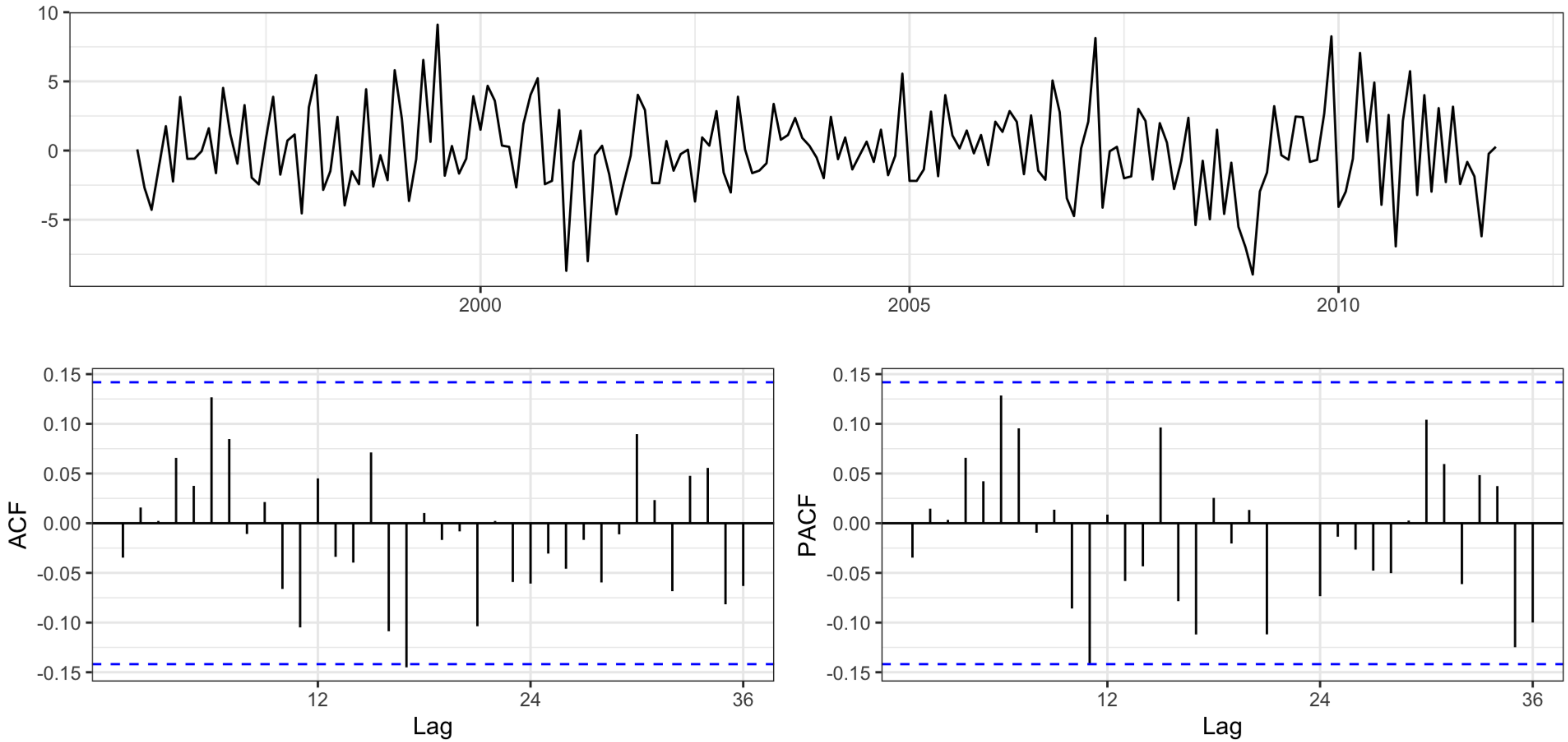
Model

```r
forecast::Arima(elec_sales, order = c(3,1,0))
```

```
Series: elec_sales
ARIMA(3,1,0)

Coefficients:
          ar1      ar2     ar3
      -0.3488  -0.0386  0.3139
s.e.   0.0690   0.0736  0.0694

sigma^2 = 9.853:  log likelihood = -485.67
AIC=979.33   AICc=979.55   BIC=992.32
```

Residuals

```r
forecast::Arima(elec_sales, order = c(3,1,0)) %>% residuals() %>%
  forecast::ggtsdisplay(points=FALSE)
```



Model Comparison

Model choices:

```
forecast::Arima(elec_sales, order = c(3,1,0))$aicc
```

```
[1] 979.5477
```

```
forecast::Arima(elec_sales, order = c(3,1,1))$aicc
```

```
[1] 978.4925
```

```
forecast::Arima(elec_sales, order = c(4,1,0))$aicc
```

```
[1] 979.2309
```

```
forecast::Arima(elec_sales, order = c(2,1,0))$aicc
```

```
[1] 996.8085
```

Automatic selection (AICc)

```r
forecast::auto.arima(elec_sales)
```

```
Series: elec_sales
ARIMA(3,1,1)

Coefficients:
         ar1     ar2     ar3      ma1
      0.0519  0.1191  0.3730  -0.4542
s.e.  0.1840  0.0888  0.0679   0.1993

sigma^2 = 9.737:  log likelihood = -484.08
AIC=978.17   AICc=978.49   BIC=994.4
```

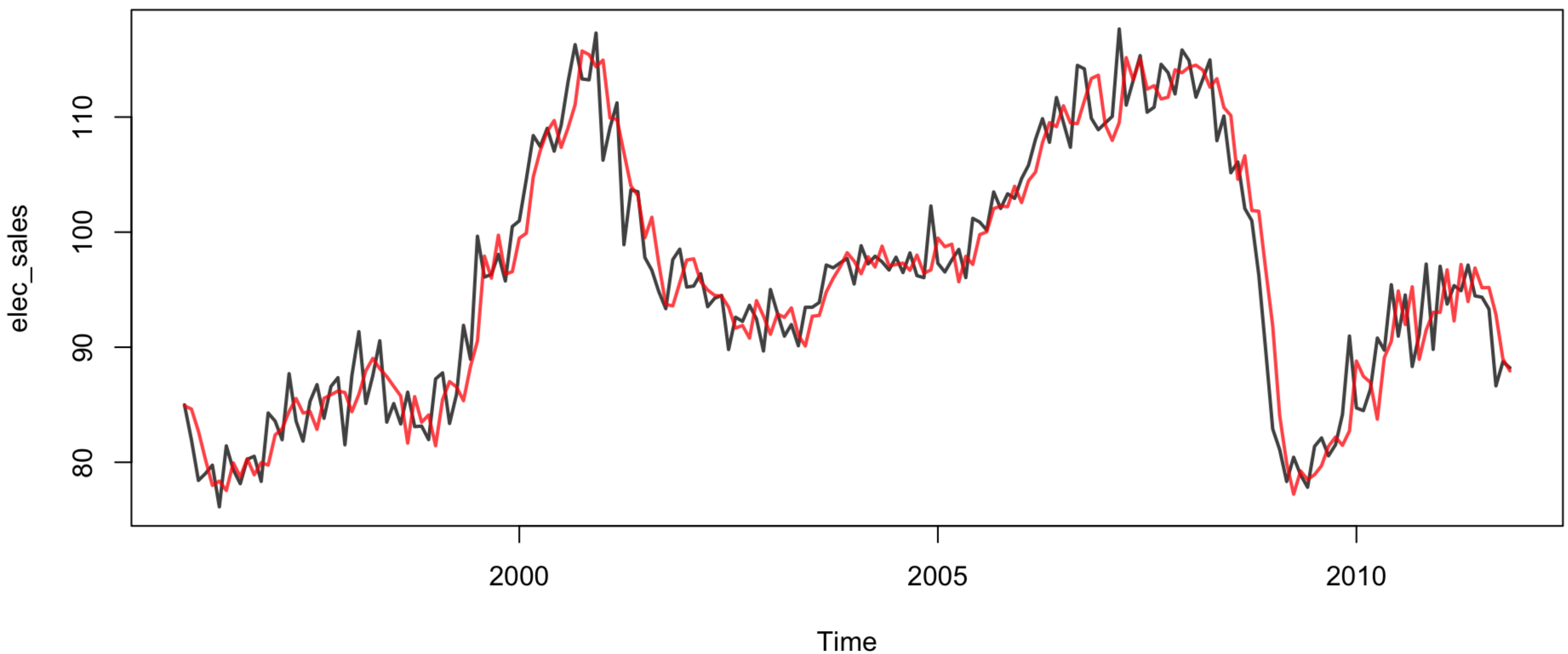
Automatic selection (BIC)

```r
forecast::auto.arima(elec_sales, ic = "bic")
```

```
Series: elec_sales 
ARIMA(1,1,2) 

Coefficients:
         ar1      ma1     ma2
      0.7738  -1.2298  0.5106
s.e.  0.0933   0.1035  0.0695

sigma^2 = 10.2:  log likelihood = -488.99
AIC=985.97   AICc=986.19   BIC=998.96
```

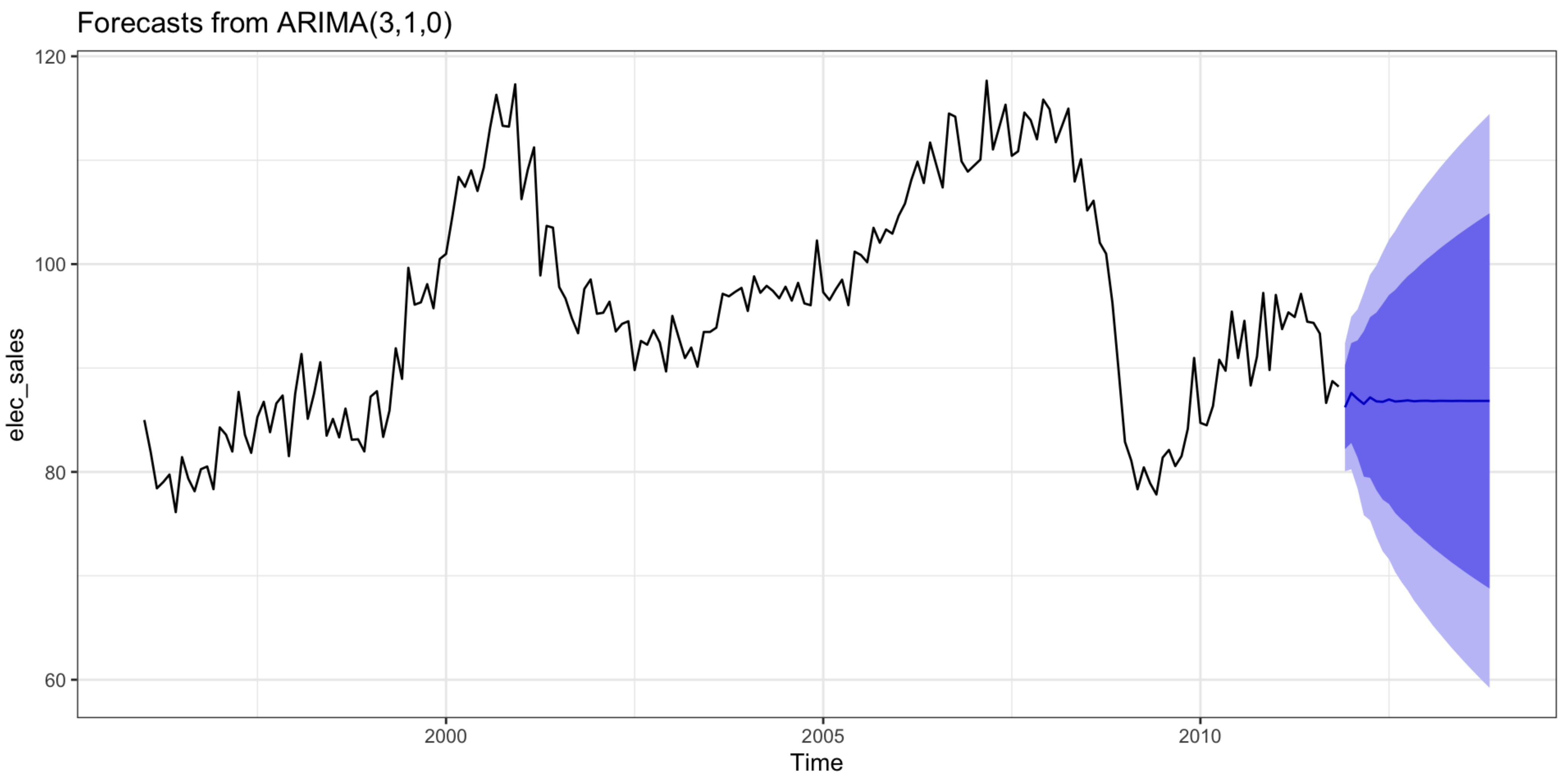
Model fit

```r
plot(elec_sales, lwd=2, col=adjustcolor("black", alpha.f=0.75))
forecast::Arima(elec_sales, order = c(3,1,0)) %>% fitted() %>%
  lines(col=adjustcolor('red',alpha.f=0.75),lwd=2)
```



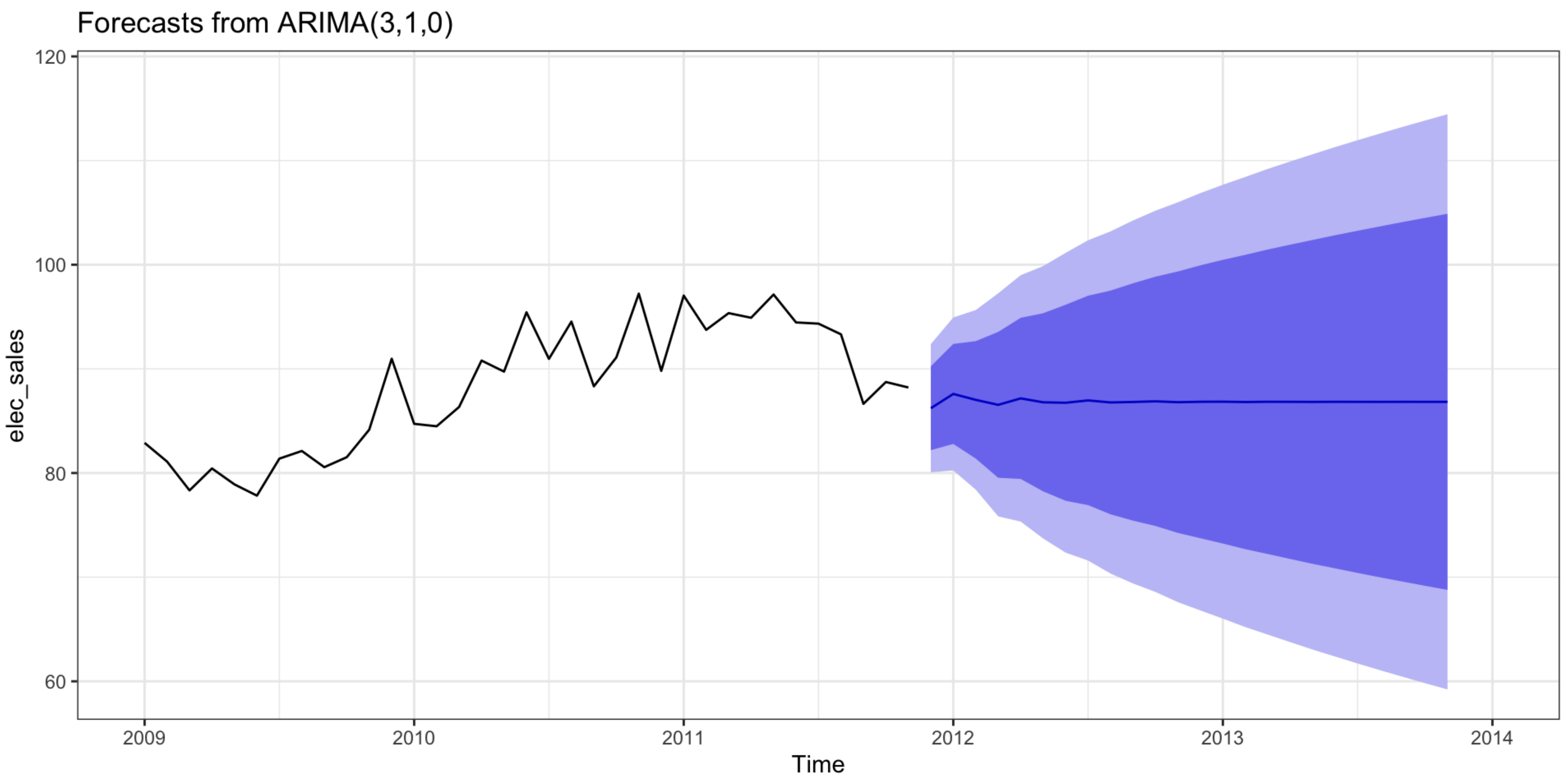
Model forecast

```r
forecast::Arima(elec_sales, order = c(3,1,0)) %>%
  forecast::forecast() %>% autoplot()
```



Model forecast - Zoom

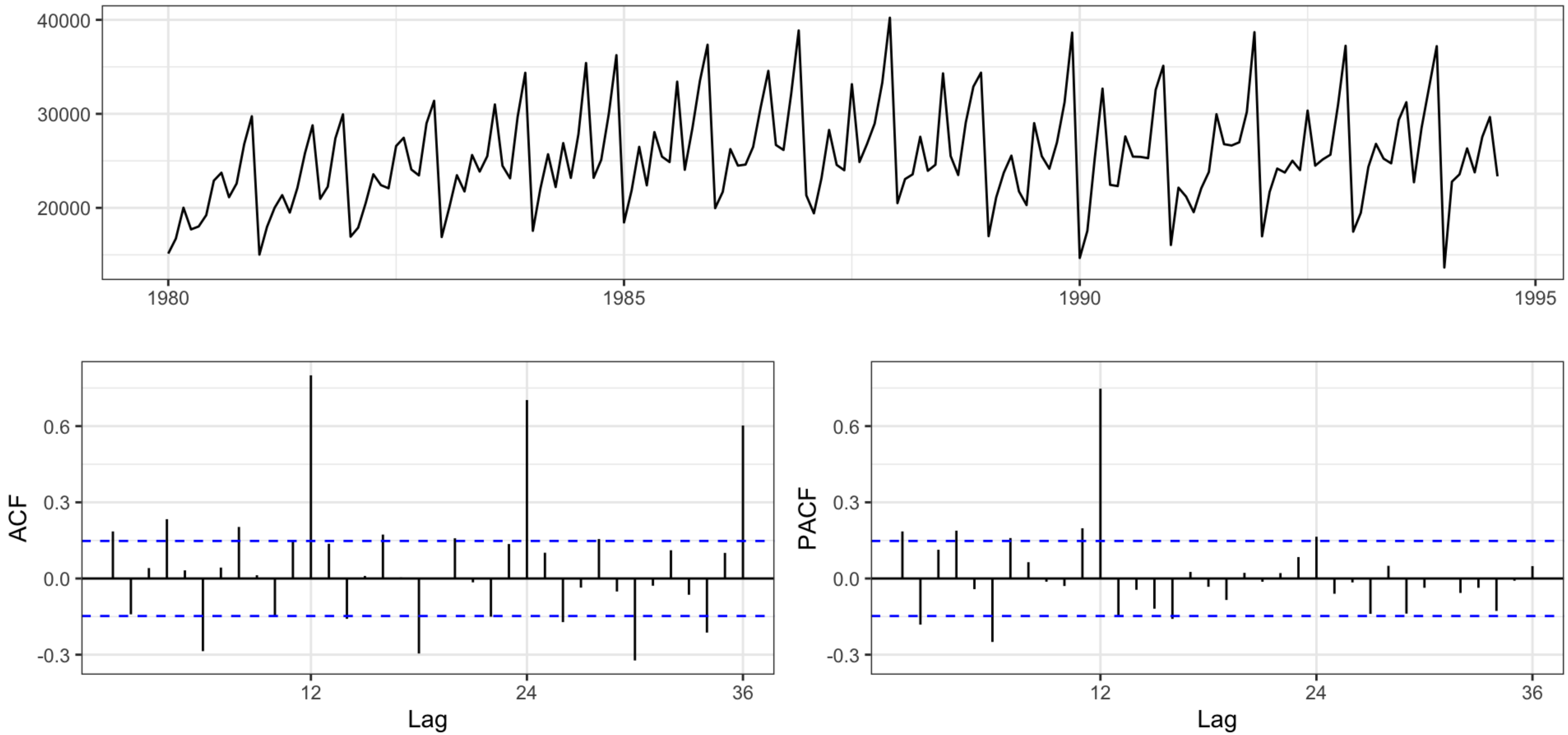
```
forecast::Arima(elec_sales, order = c(3,1,0)) %>%
  forecast::forecast() %>% autoplot() + xlim(2009,2014)
```



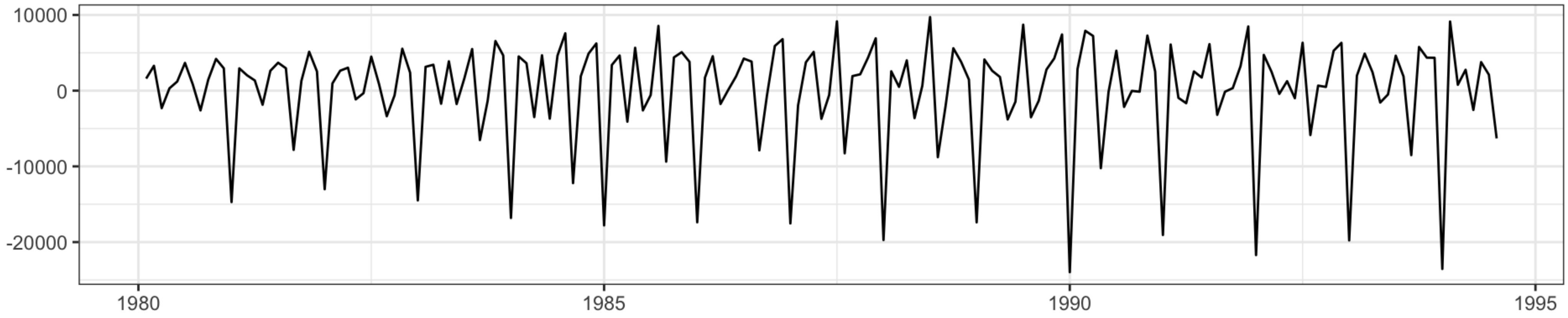
Seasonal Models

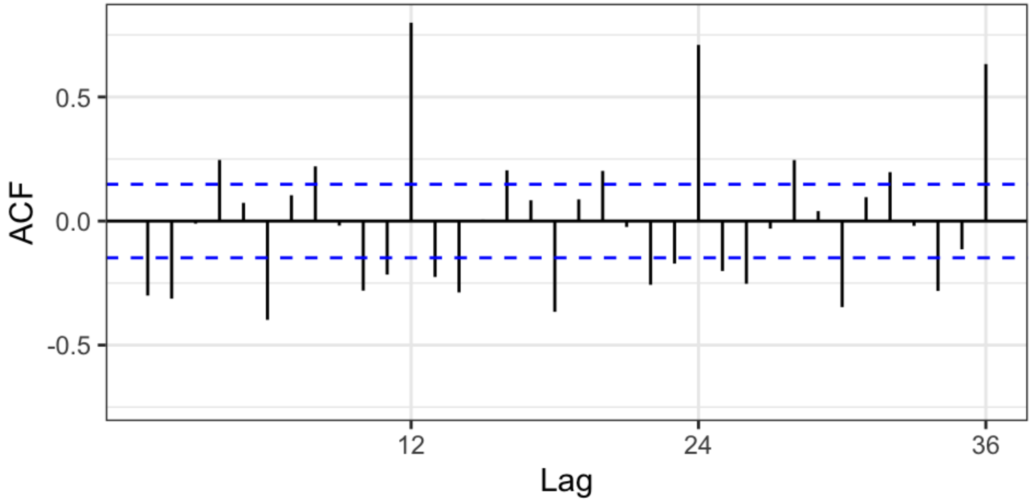
Australian Wine Sales Example

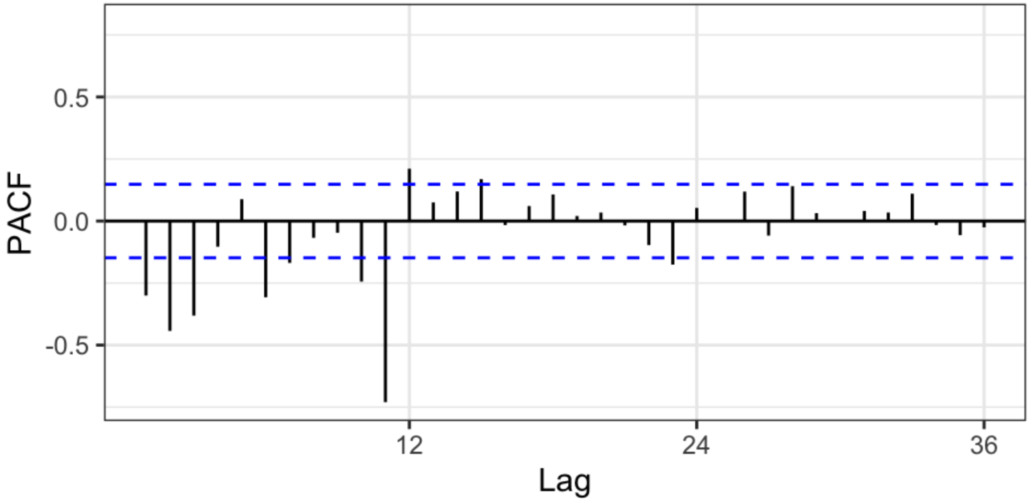
Australian total wine sales by wine makers in bottles <= 1 litre. Jan 1980 – Aug 1994.



Differencing







Seasonal Arima

We can extend the existing ARIMA model to handle these higher order lags (without having to include all of the intervening lags).

Seasonal $\text{ARIMA}\,(p, d, q) \times (P, D, Q)_s:$

$$\Phi_P(L^s)\,\phi_p(L)\,\Delta_s^D\,\Delta^d\,y_t = \delta + \Theta_Q(L^s)\,\theta_q(L)\,w_t$$

. . .

where

$$\begin{aligned}
\phi_p(L) &= 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p \\
\theta_q(L) &= 1 + \theta_1 L + \theta_2 L^2 + \ldots + \theta_p L^q \\
\Delta^d &= (1 - L)^d
\end{aligned}$$

$$\begin{aligned}
\Phi_P(L^s) &= 1 - \Phi_1 L^s - \Phi_2 L^{2s} - \ldots - \Phi_P L^{Ps} \\
\Theta_Q(L^s) &= 1 + \Theta_1 L + \Theta_2 L^{2s} + \ldots + \theta_p L^{Qs} \\
\Delta_s^D &= (1 - L^s)^D
\end{aligned}$$

Seasonal ARIMA - AR

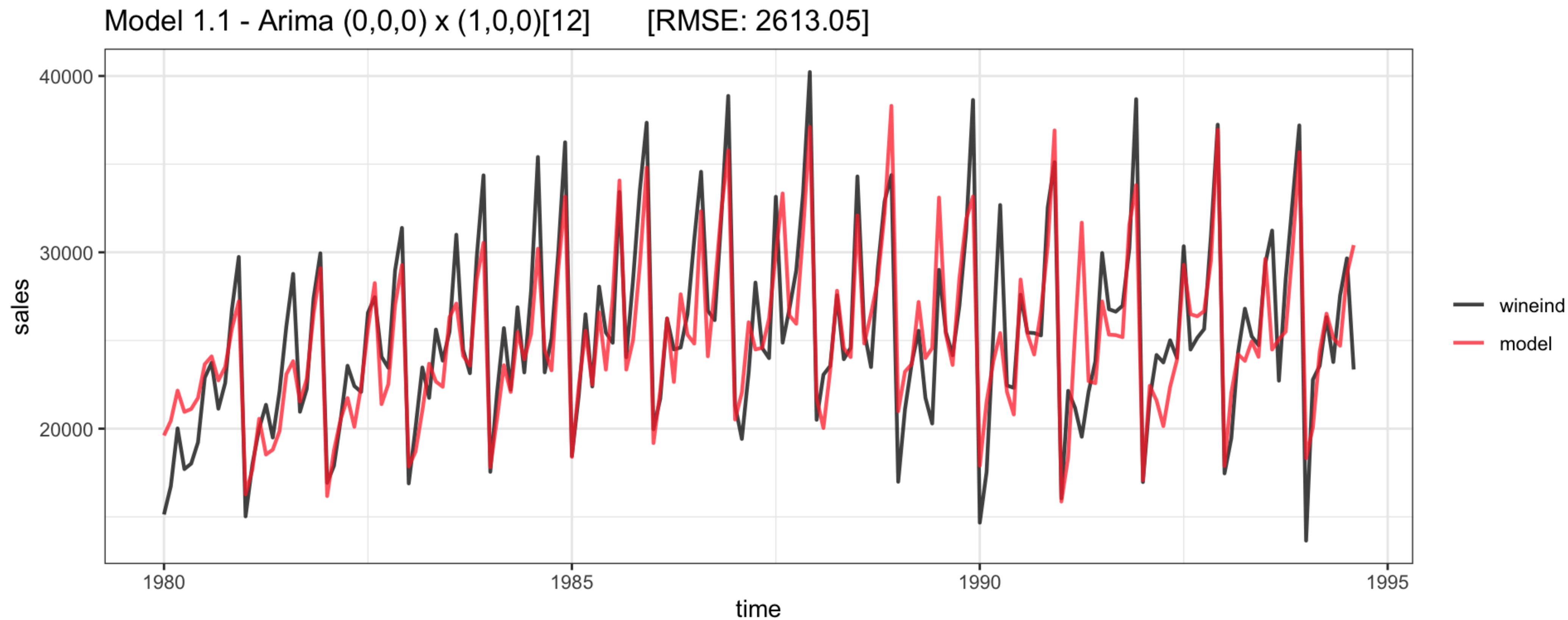
Lets consider an $\text{ARIMA}(0,0,0) \times (1,0,0)_{12}$:

$$(1 - \Phi_1 L^{12})\, y_t = \delta + w_t$$
$$y_t = \Phi_1 y_{t-12} + \delta + w_t$$

```
(m1.1 = forecast::Arima(wineind, seasonal=list(order=c(1,0,0), period=12
```

```
Series: wineind
ARIMA(0,0,0)(1,0,0)[12] with non-zero mean

Coefficients:
        sar1       mean
      0.8780  24489.243
s.e.  0.0314   1154.487

sigma^2 = 6906536:  log likelihood = -1643.39
AIC=3292.78   AICc=3292.92   BIC=3302.29
```

Fitted - Model 1.1



Seasonal Arima - Diff

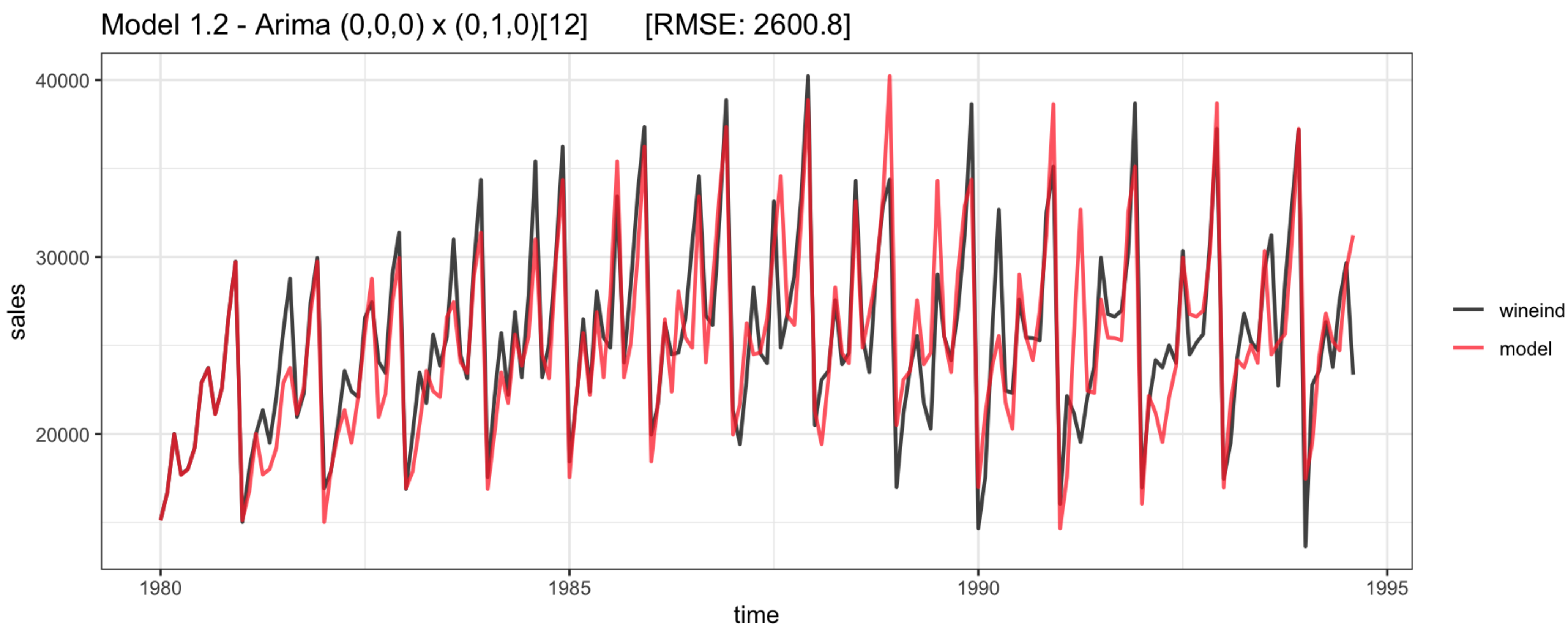
Lets consider an $\text{ARIMA}(0,0,0) \times (0,1,0)_{12}$:

$$
\begin{aligned}
(1 - L^{12})\, y_t &= \delta + w_t \\
y_t &= y_{t-12} + \delta + w_t
\end{aligned}
$$

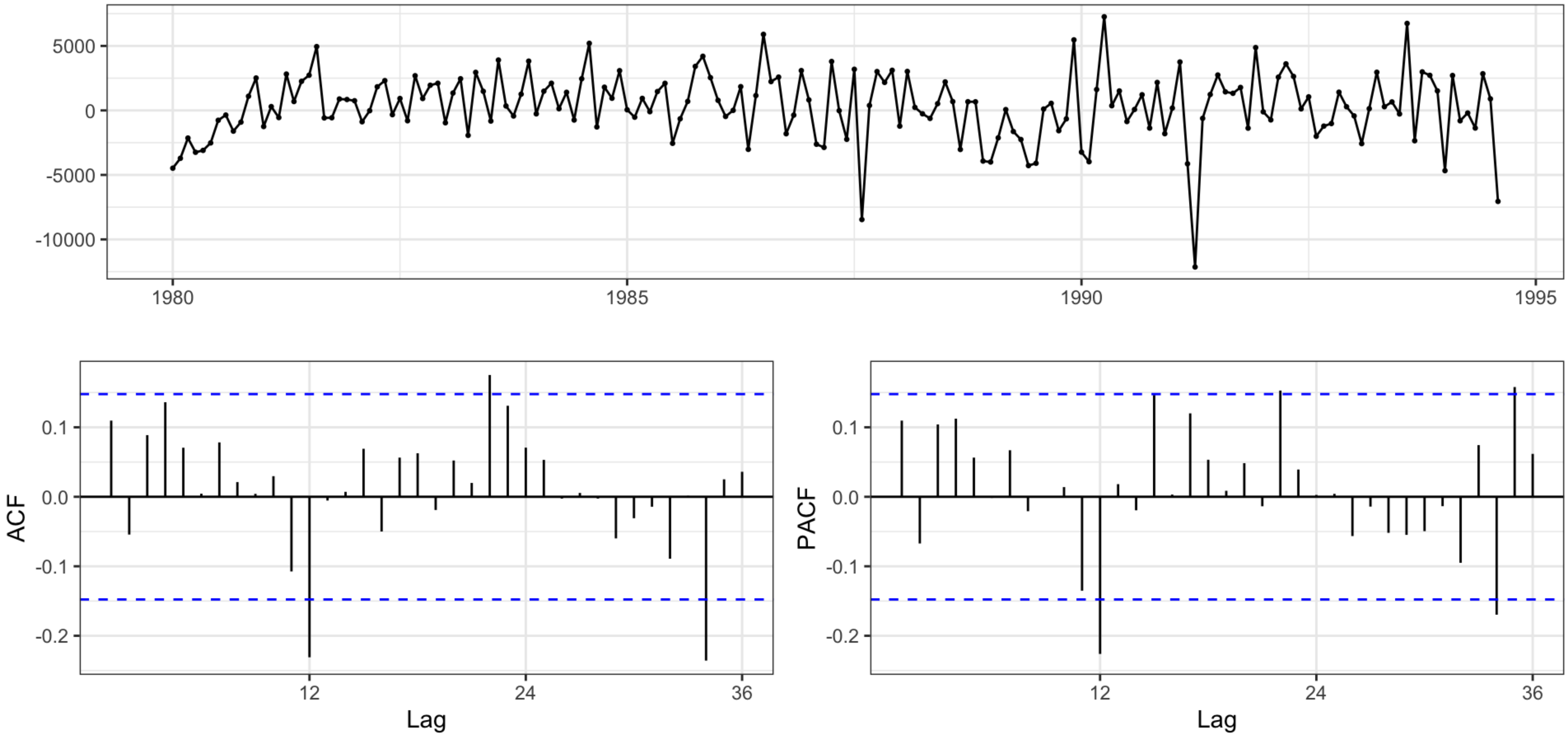
```r
(m1.2 = forecast::Arima(
  wineind, seasonal=list(order=c(0,1,0), period=12)
))
```

```
Series: wineind 
ARIMA(0,0,0)(0,1,0)[12] 

sigma^2 = 7259076:  log likelihood = -1528.12
AIC=3058.24   AICc=3058.27   BIC=3061.34
```

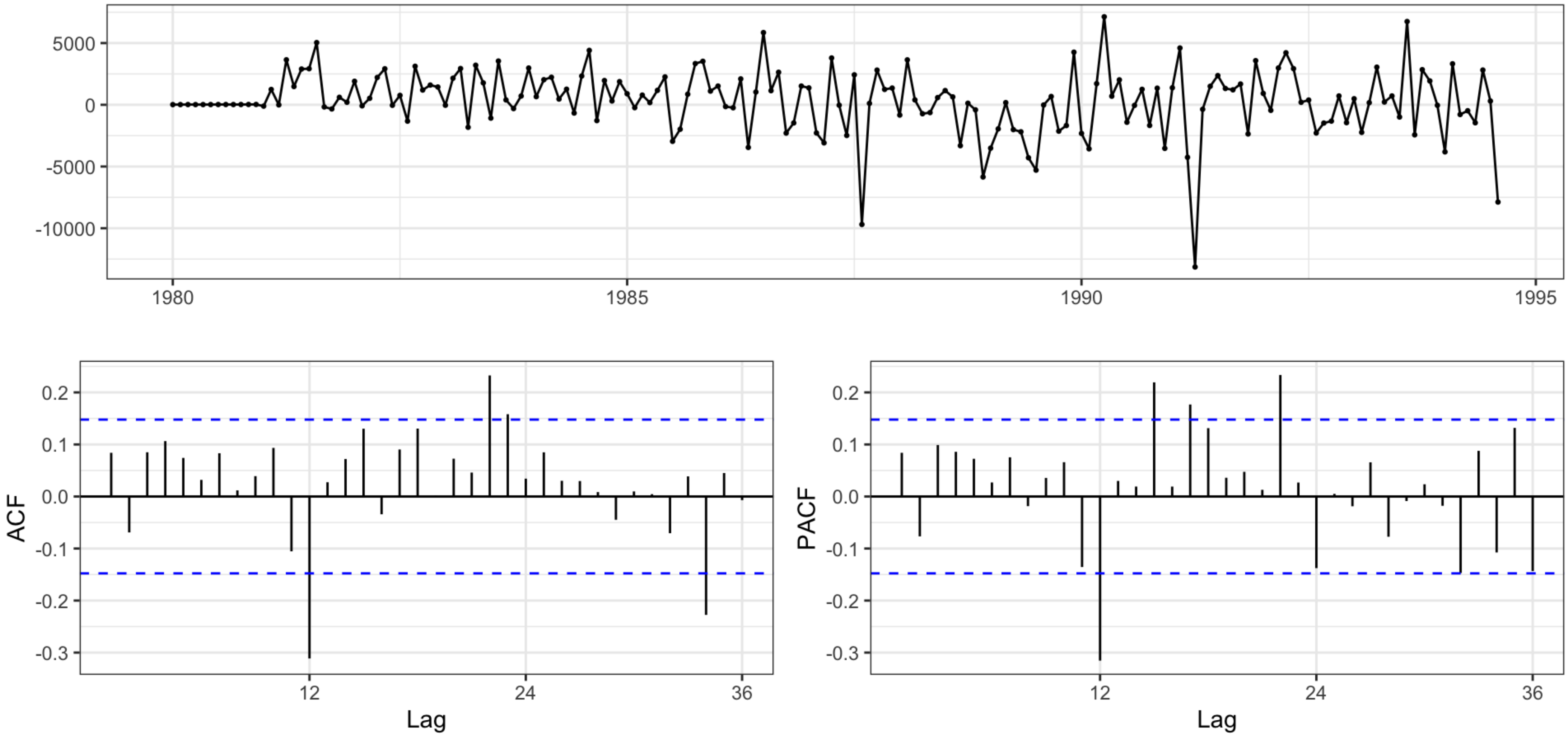
Fitted - Model 1.2



Residuals - Model 1.1 (SAR)



Residuals - Model 1.2 (SDiff)



Model 2

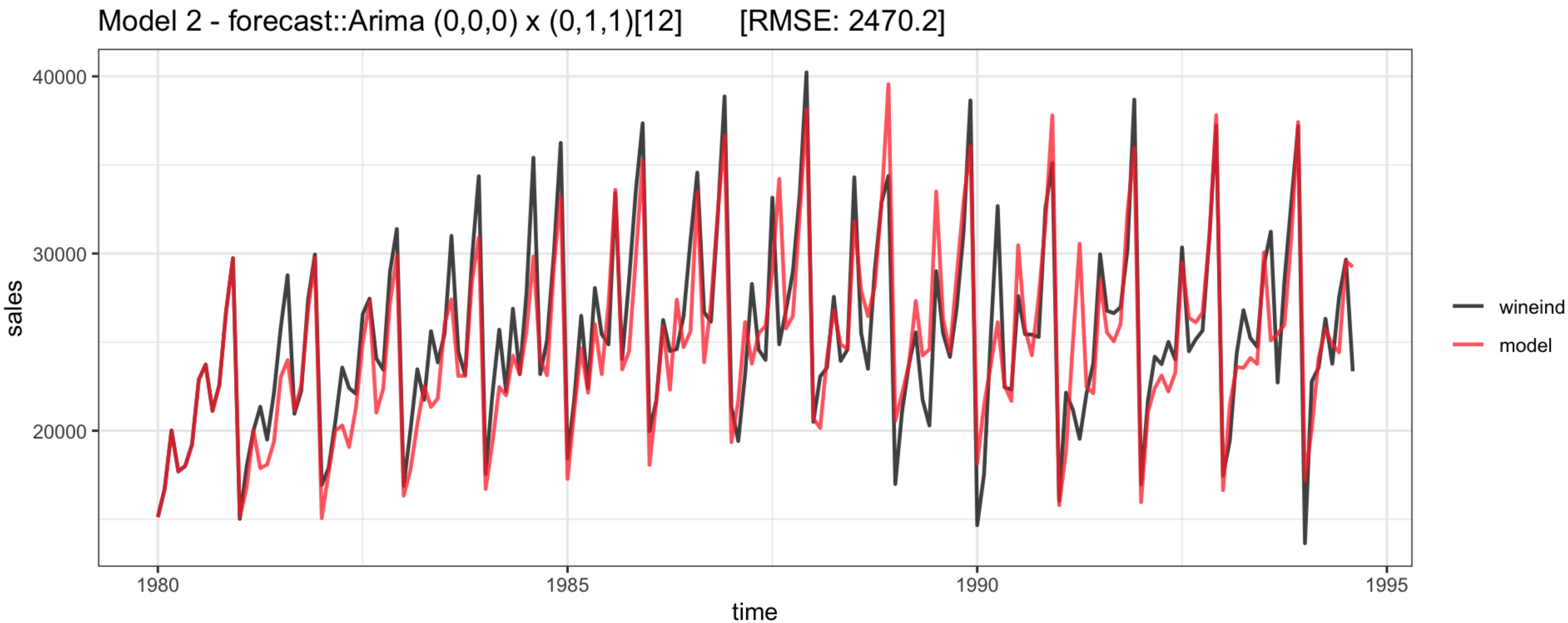
$\text{ARIMA}(0,0,0) \times (0,1,1)_{12}:$

$$(1 - L^{12})y_t = \delta + (1 + \Theta_1 L^{12})w_t$$
$$y_t = \delta + y_{t-12} + w_t + \Theta_1 w_{t-12}$$

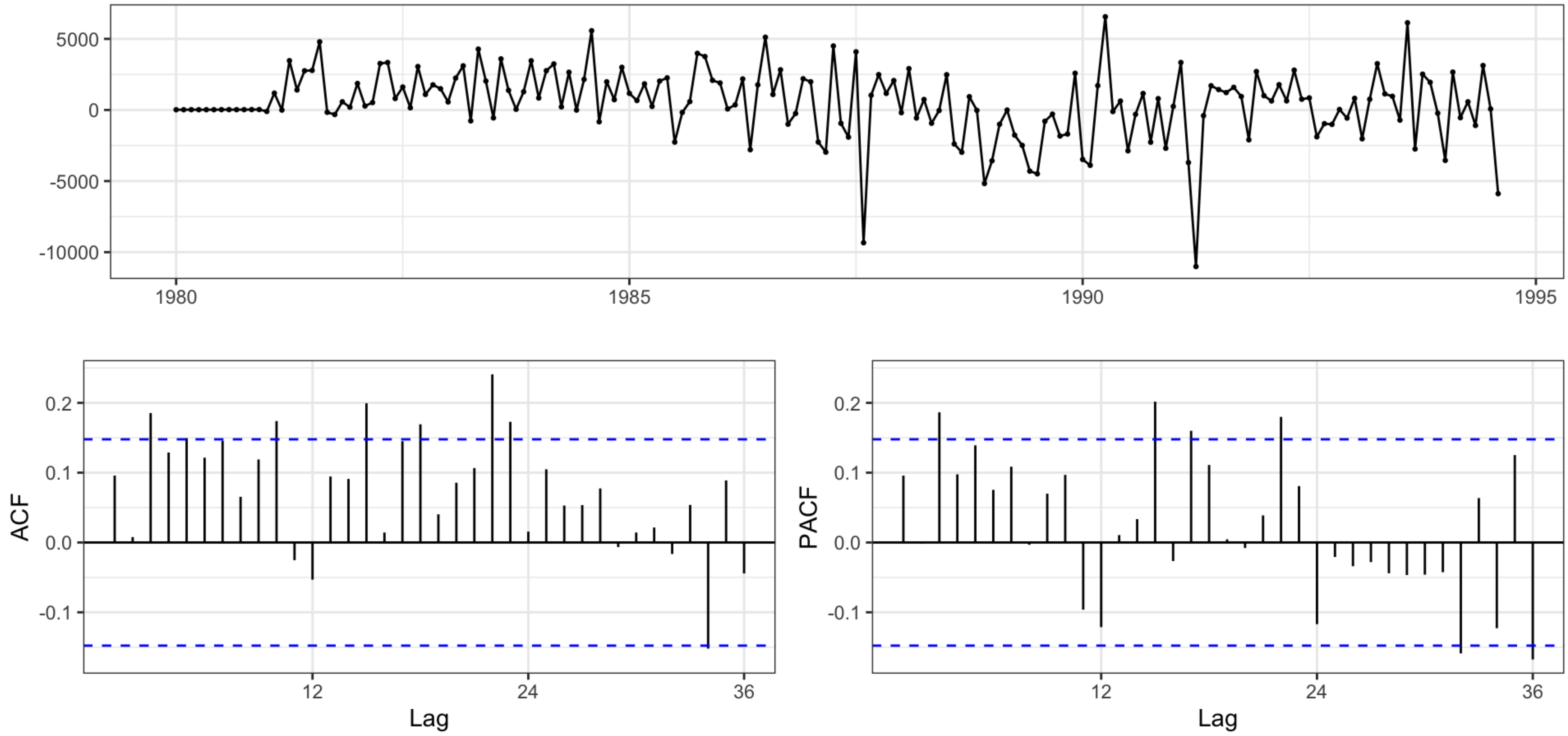
```r
(m2 = forecast::Arima(wineind, order=c(0,0,0),
                      seasonal=list(order=c(0,1,1), period=12)))
```

```
Series: wineind
ARIMA(0,0,0)(0,1,1)[12]

Coefficients:
         sma1
      -0.3246
s.e.   0.0807

sigma^2 = 6588531:  log likelihood = -1520.34
AIC=3044.68   AICc=3044.76   BIC=3050.88
```

Fitted - Model 2



Residuals


5000
0
-5000
-10000
1980
1985
1990
1995
0.2
0.1
0.0
-0.1
ACF
12
24
36
Lag
0.2
0.1
0.0
-0.1
PACF
12
24
36
Lag

Model 3

$\text{ARIMA}(3,0,0) \times (0,1,1)_{12}$

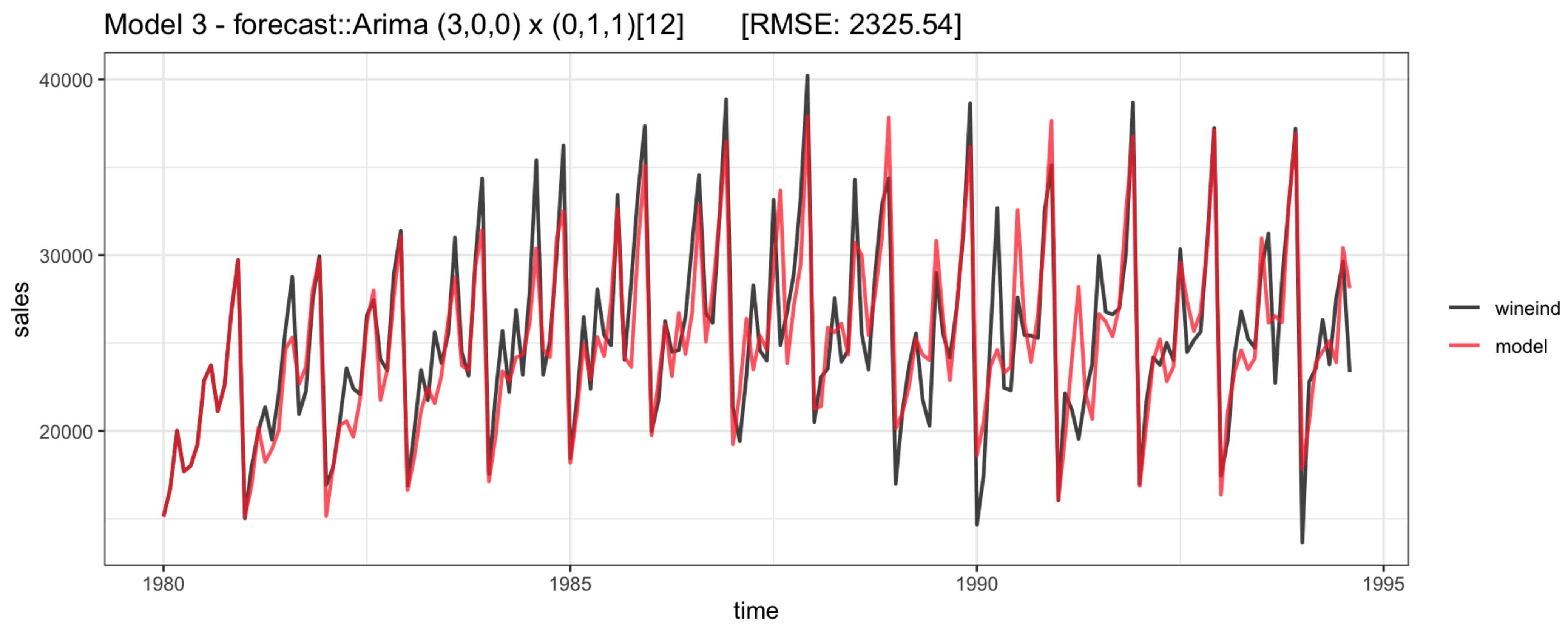
$$(1 - \phi_1 L - \phi_2 L^2 - \phi_3 L^3)\,(1 - L^{12}) y_t = \delta + (1 + \Theta_1 L) w_t$$

$$y_t = \delta + \sum_{i=1}^{3} \phi_i y_{t-i} + y_{t-12} - \sum_{i=1}^{3} \phi_i y_{t-12-i} + w_t + w_{t-12}$$

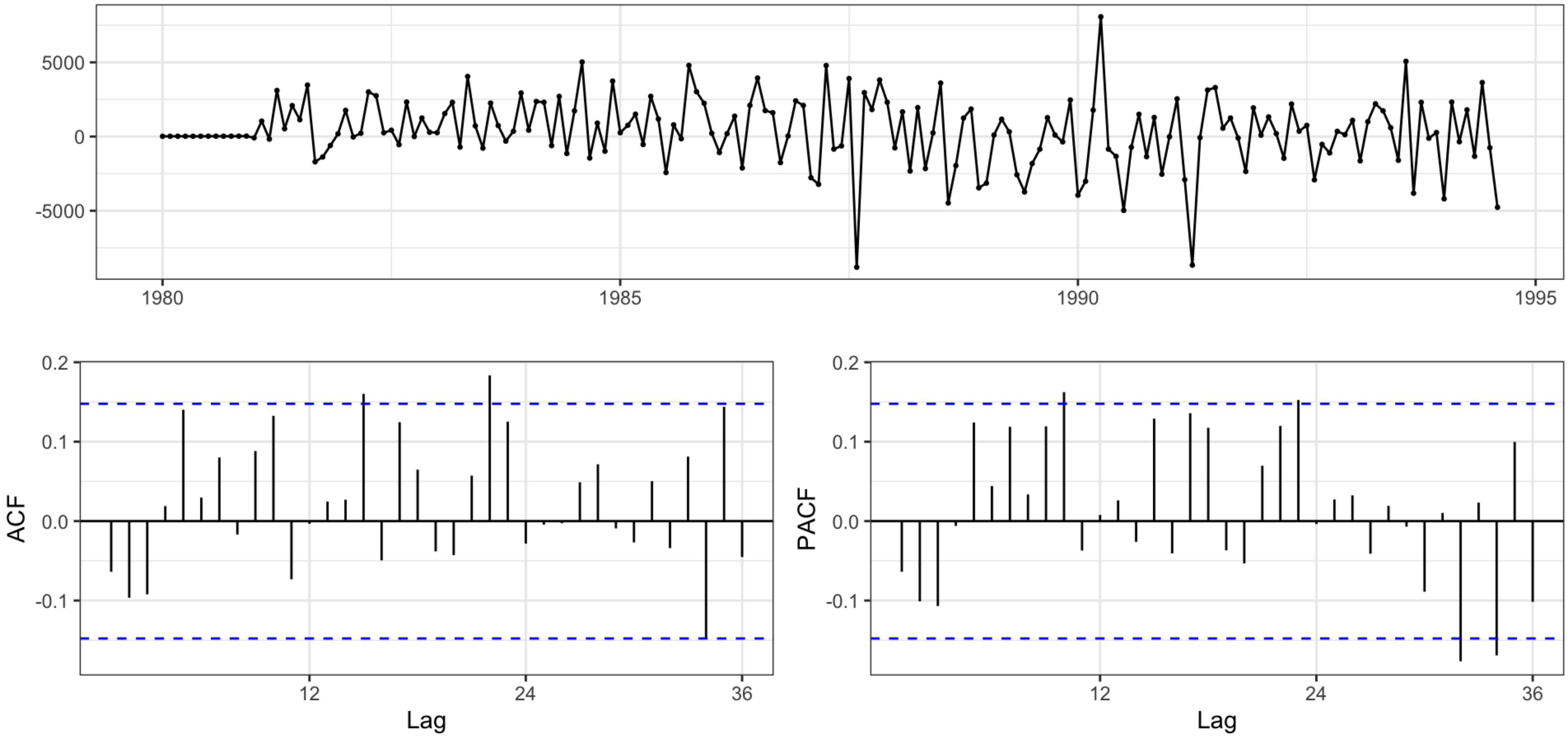
```r
(m3 = forecast::Arima(wineind, order=c(3,0,0),
                      seasonal=list(order=c(0,1,1), period=12)))
```

```
Series: wineind
ARIMA(3,0,0)(0,1,1)[12]

Coefficients:
         ar1     ar2     ar3     sma1
      0.1402  0.0806  0.3040  -0.5790
s.e.  0.0755  0.0813  0.0823   0.1023

sigma^2 = 5948935:  log likelihood = -1512.38
AIC=3034.77   AICc=3035.15   BIC=3050.27
```

Fitted model



Model - Residuals

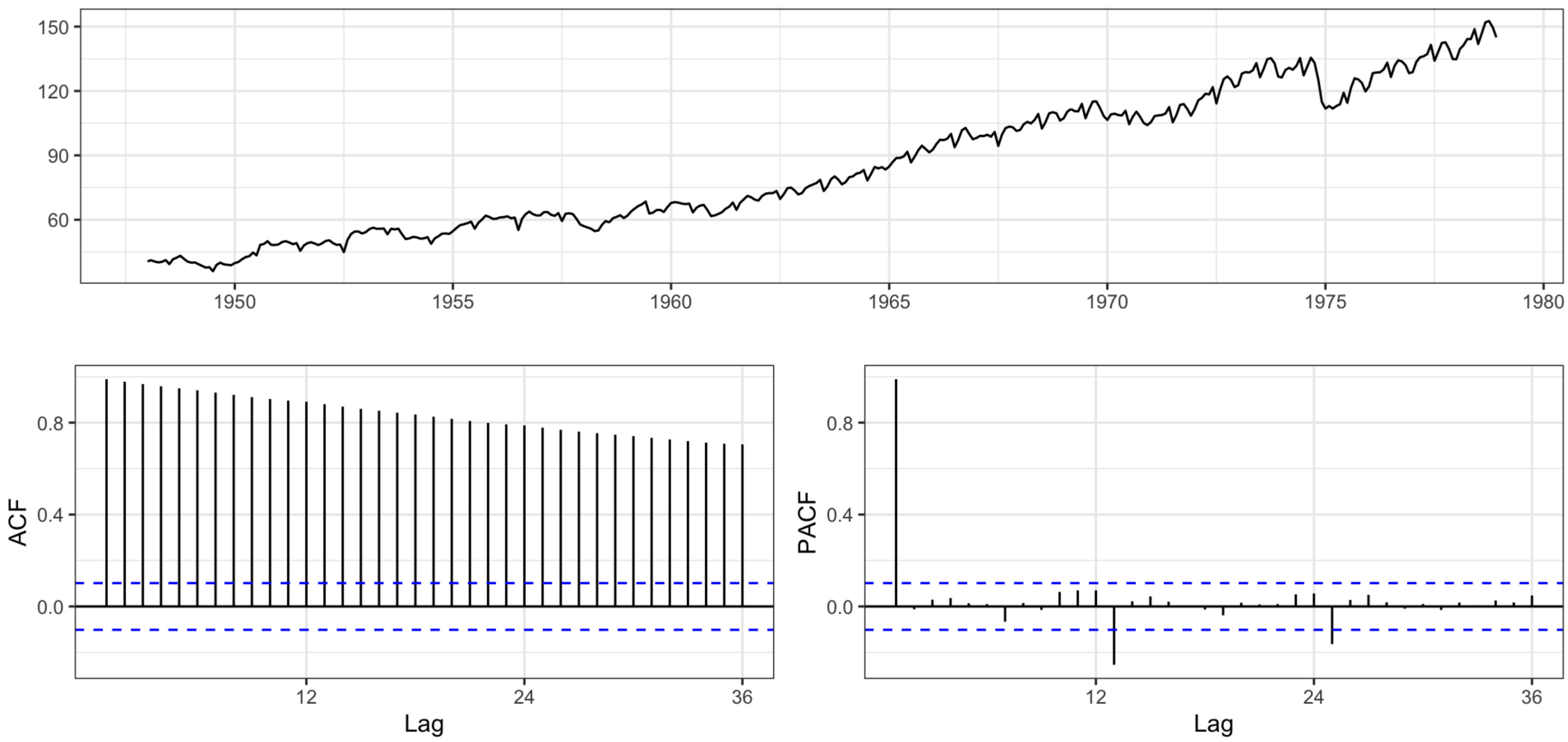


Federal Reserve Board Production Index

`prodn` from the `astsa` package

Monthly Federal Reserve Board Production Index (1948-1978)

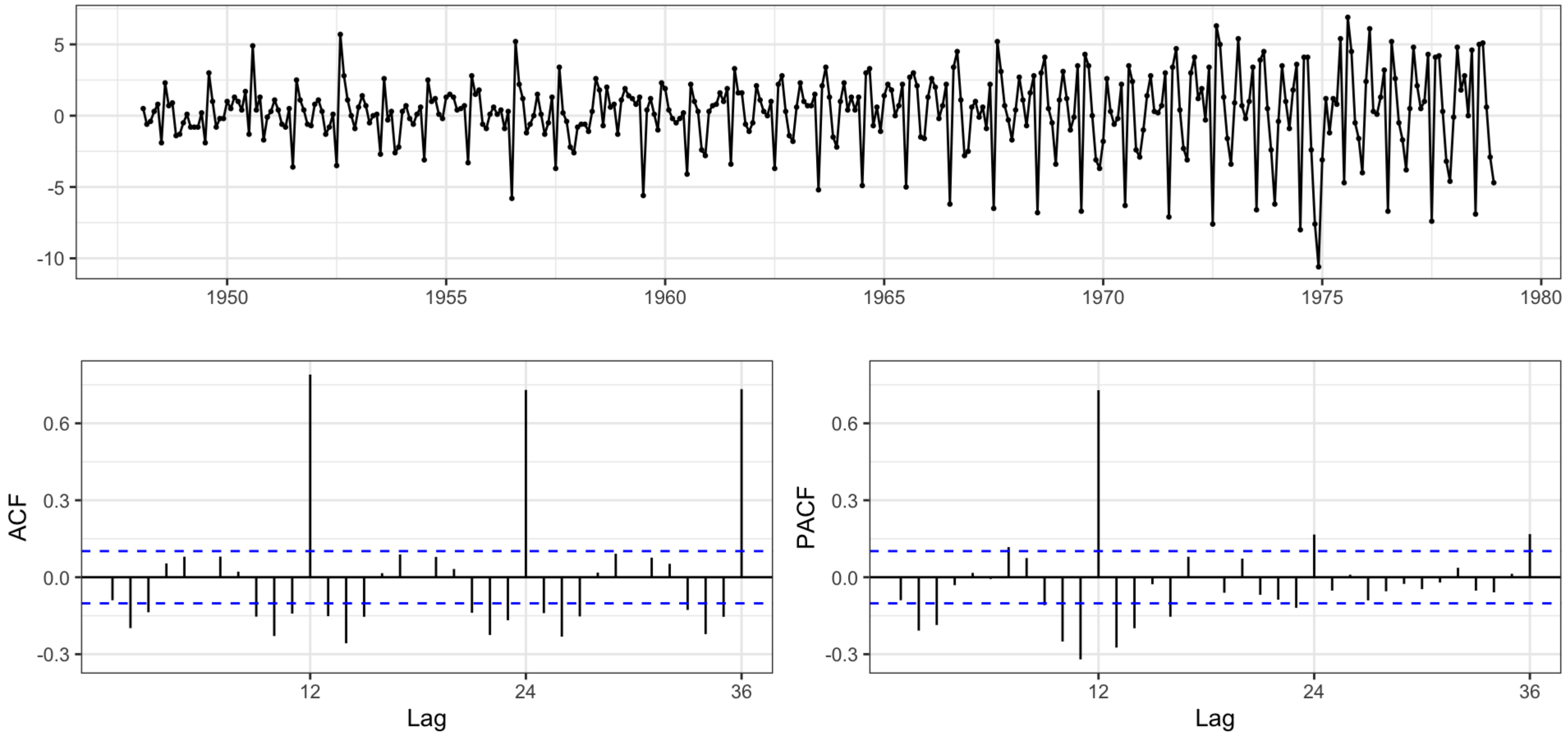
```r
data(prodn, package="astsa")
forecast::ggtsdisplay(prodn, points = FALSE)
```



Differencing

Based on the ACF it seems like standard differencing may be required

```
forecast::ggtsdisplay(diff(prodn))
```



Differencing + Seasonal Differencing

Additional seasonal differencing also seems warranted

```r
(fr_m1 = forecast::Arima(
  prodn, order = c(0,1,0),
  seasonal = list(order=c(0,0,0), period=12)
))
```

```
Series: prodn
ARIMA(0,1,0)

sigma^2 = 7.147:  log likelihood = -891.26
AIC=1784.51   AICc=1784.52   BIC=1788.43
```

```r
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m1$fitted %>% unclass()
)
```

```
[1] 2.669854
```

```r
(fr_m2 = forecast::Arima(
  prodn, order = c(0,1,0),
  seasonal = list(order=c(0,1,0), period=12)
))
```

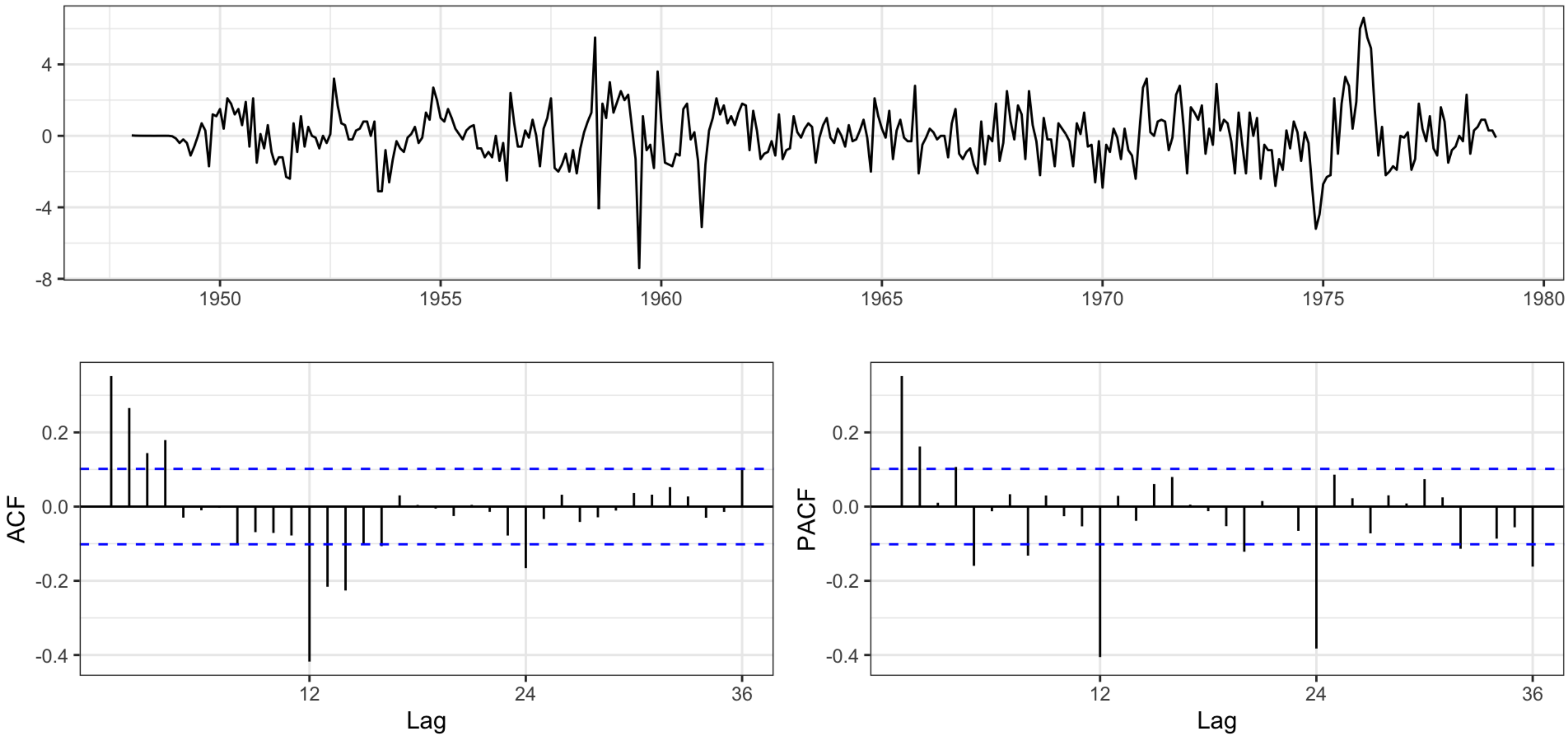
```
Series: prodn
ARIMA(0,1,0)(0,1,0)[12]

sigma^2 = 2.52:  log likelihood = -675.29
AIC=1352.58   AICc=1352.59   BIC=1356.46
```

```r
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m2$fitted %>% unclass()
)
```

```
[1] 1.559426
```

Residuals - Model 2

```r
forecast::ggtsdisplay(fr_m2$residuals, points=FALSE, lag.max=36)
```



Adding Seasonal MA

```r
(fr_m3.1 = forecast::Arima(
  prodn, order = c(0,1,0),
  seasonal = list(order=c(0,1,1), period=12)
))
```

```
Series: prodn
ARIMA(0,1,0)(0,1,1)[12]

Coefficients:
         sma1
      -0.7151
s.e.   0.0317

sigma^2 = 1.616:  log likelihood = -599.29
AIC=1202.57   AICc=1202.61   BIC=1210.34
```

```r
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m3.1$fitted %>% unclass()
)
```

```
[1] 1.246885
```

```r
(fr_m3.2 = forecast::Arima(
  prodn, order = c(0,1,0),
  seasonal = list(order=c(0,1,2), period=12)
))
```

```
Series: prodn
ARIMA(0,1,0)(0,1,2)[12]

Coefficients:
         sma1    sma2
      -0.7624  0.0520
s.e.   0.0689  0.0666

sigma^2 = 1.615:  log likelihood = -598.98
AIC=1203.96   AICc=1204.02   BIC=1215.61
```

```r
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m3.2$fitted %>% unclass()
)
```

```
[1] 1.245104
```

Adding Seasonal MA (cont.)

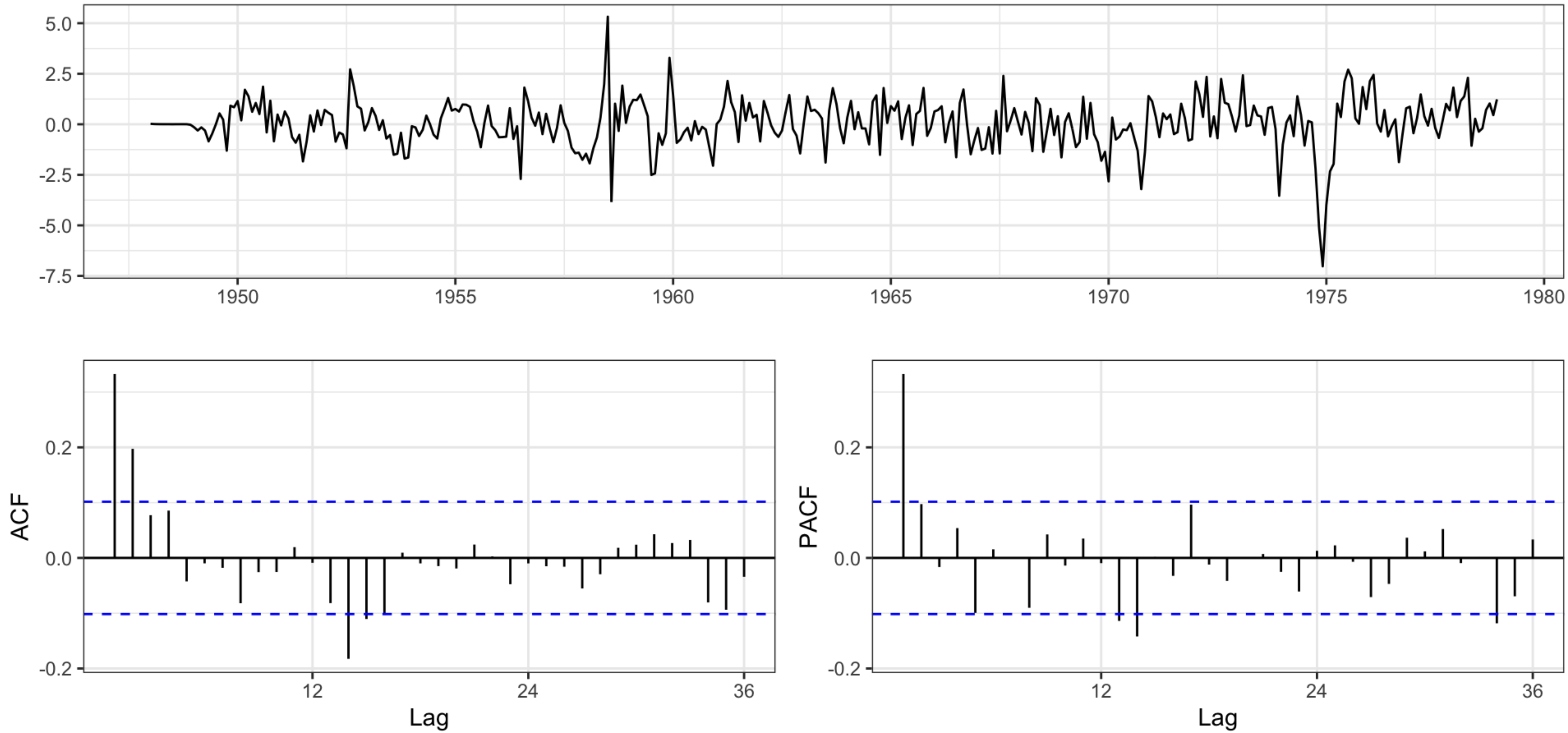
```
(fr_m3.3 = forecast::Arima(
  prodn, order = c(0,1,0),
  seasonal = list(order=c(0,1,3), period=12)
))
```

```
Series: prodn
ARIMA(0,1,0)(0,1,3)[12]

Coefficients:
         sma1     sma2    sma3
      -0.7853  -0.1205  0.2624
s.e.   0.0529   0.0644  0.0529

sigma^2 = 1.506:  log likelihood = -587.58
AIC=1183.15   AICc=1183.27   BIC=1198.69
```

```
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m3.3$fitted %>% unclass()
)
```

```
[1] 1.200592
```

Residuals - Model 3.3



Adding AR

```r
(fr_m4.1 = forecast::Arima(
  prodn, order = c(1,1,0),
  seasonal = list(order=c(0,1,3), period=12)
))
```

```
Series: prodn
ARIMA(1,1,0)(0,1,3)[12]

Coefficients:
         ar1     sma1     sma2    sma3
      0.3393  -0.7619  -0.1222  0.2756
s.e.  0.0500   0.0527   0.0646  0.0525

sigma^2 = 1.341:  log likelihood = -565.98
AIC=1141.95   AICc=1142.12   BIC=1161.37
```

```r
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m4.1$fitted %>% unclass()
)
```

```
[1] 1.131115
```

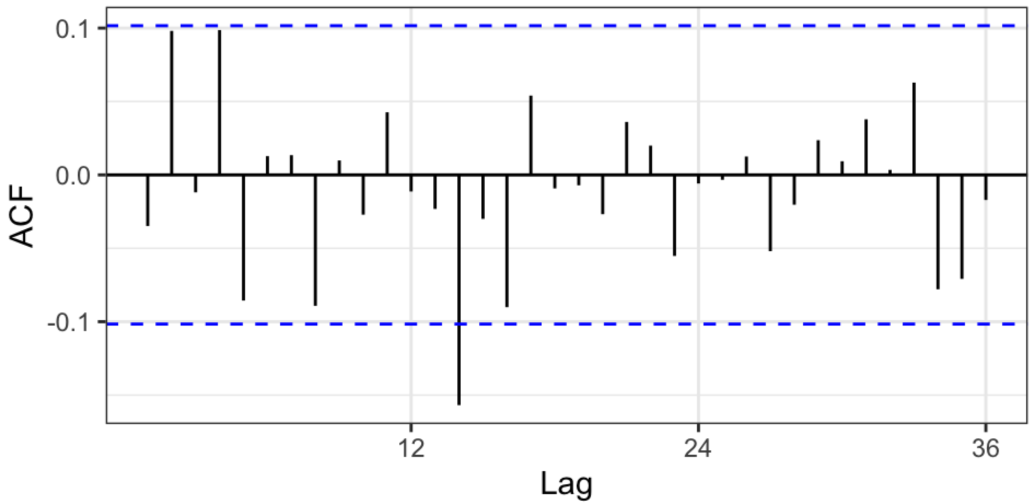
```r
(fr_m4.2 = forecast::Arima(
  prodn, order = c(2,1,0),
  seasonal = list(order=c(0,1,3), period=12)
))
```

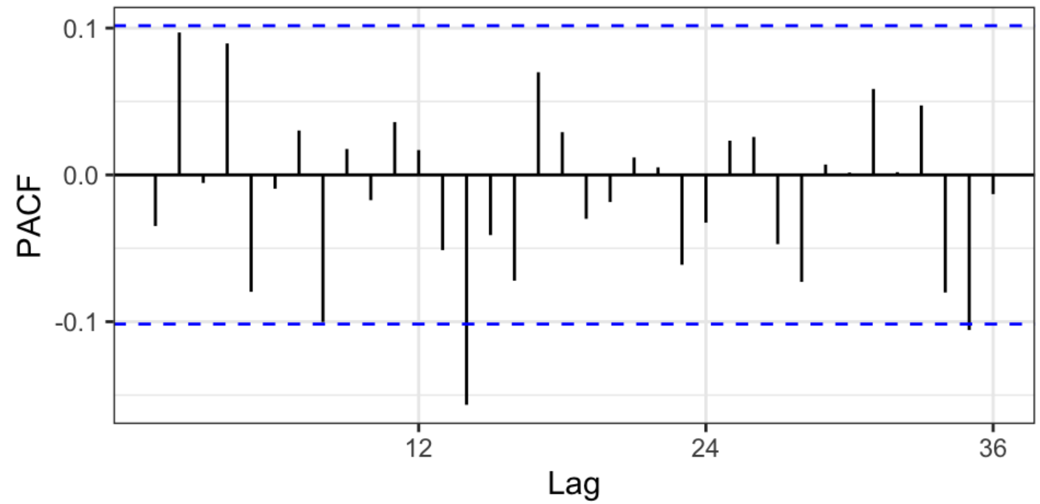
```
Series: prodn
ARIMA(2,1,0)(0,1,3)[12]

Coefficients:
         ar1     ar2     sma1     sma2    sma3
      0.3038  0.1077  -0.7393  -0.1445  0.2815
s.e.  0.0526  0.0538   0.0539   0.0653  0.0526

sigma^2 = 1.331:  log likelihood = -563.98
AIC=1139.97   AICc=1140.2   BIC=1163.26
```

```r
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m4.2$fitted %>% unclass()
)
```

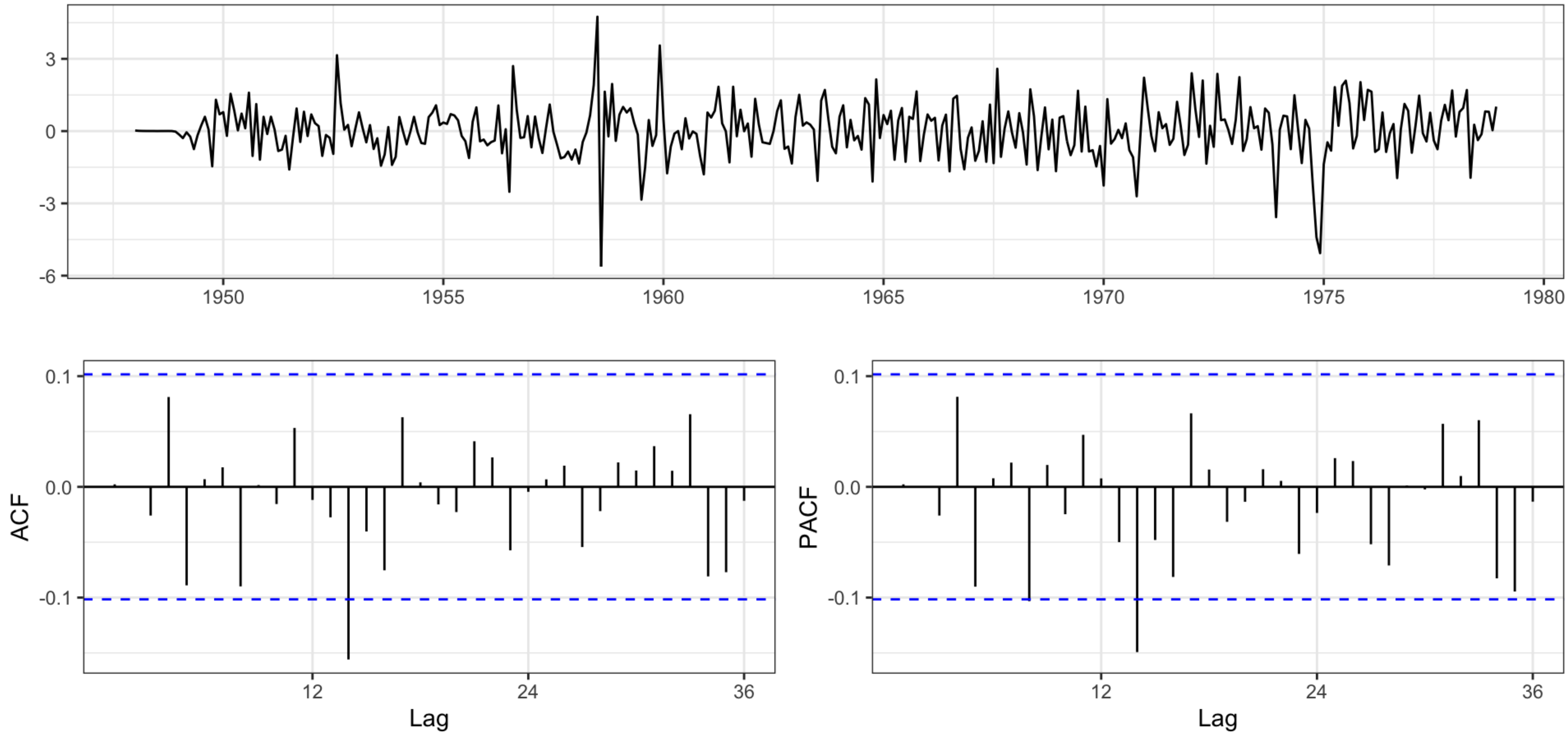
```
[1] 1.125322
```

Residuals - Model 4.1

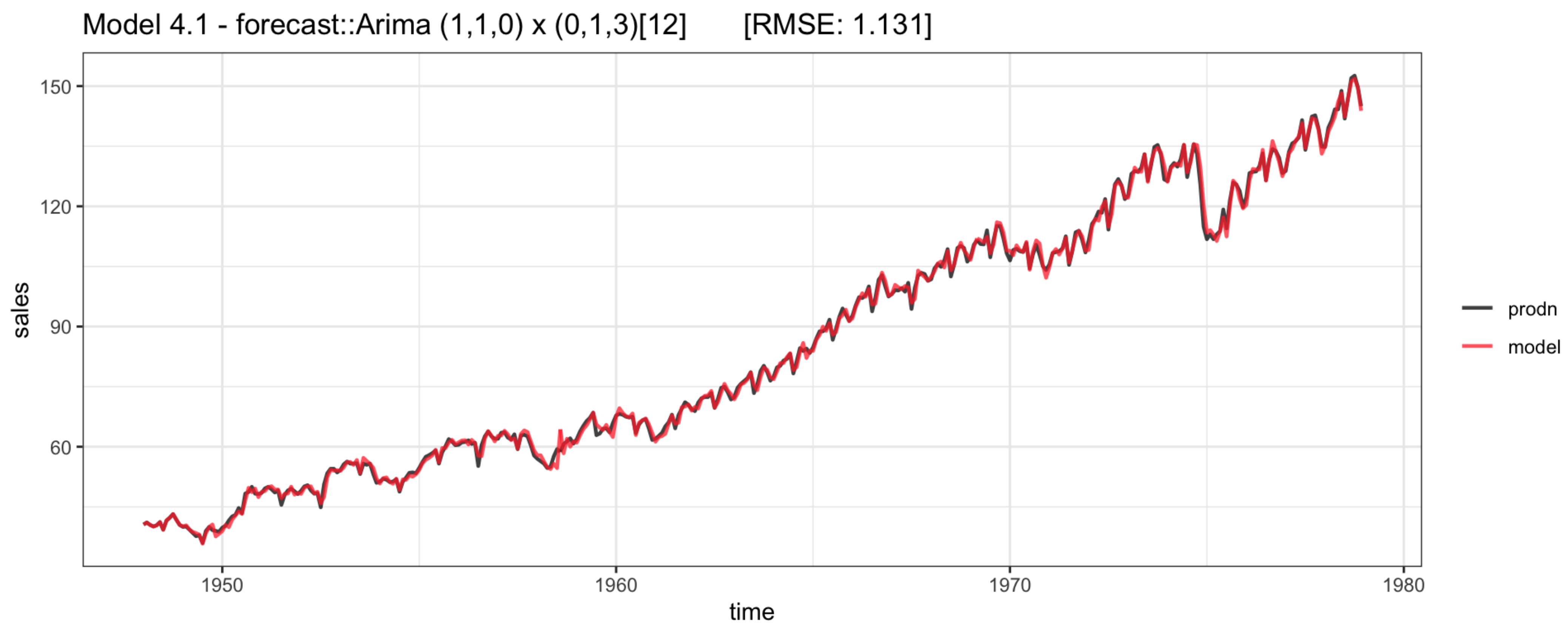




Residuals - Model 4.2



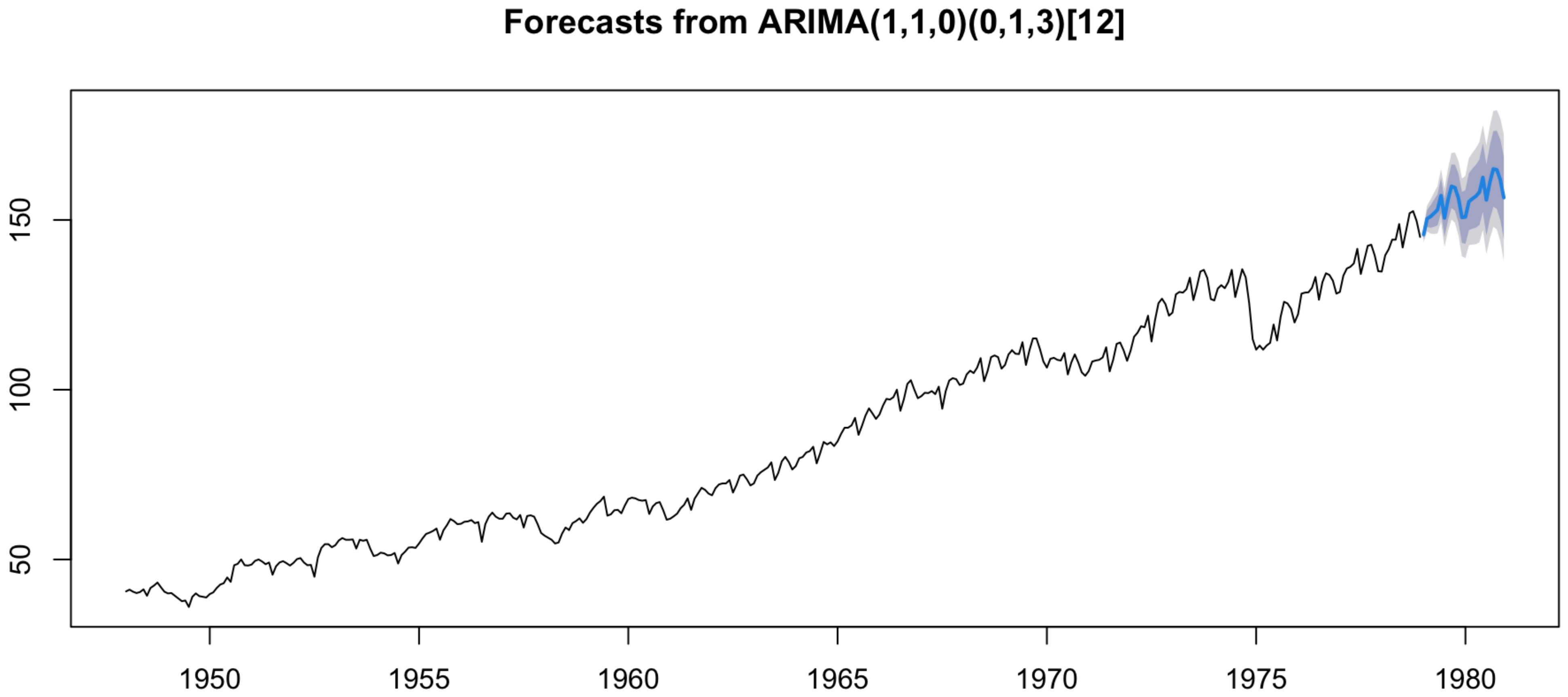
Model Fit



Model 4.1 - forecast::Arima (1,1,0) x (0,1,3)[12]   [RMSE: 1.131]
150
120
90
60
sales
1950
1960
1970
1980
time
prodn
model

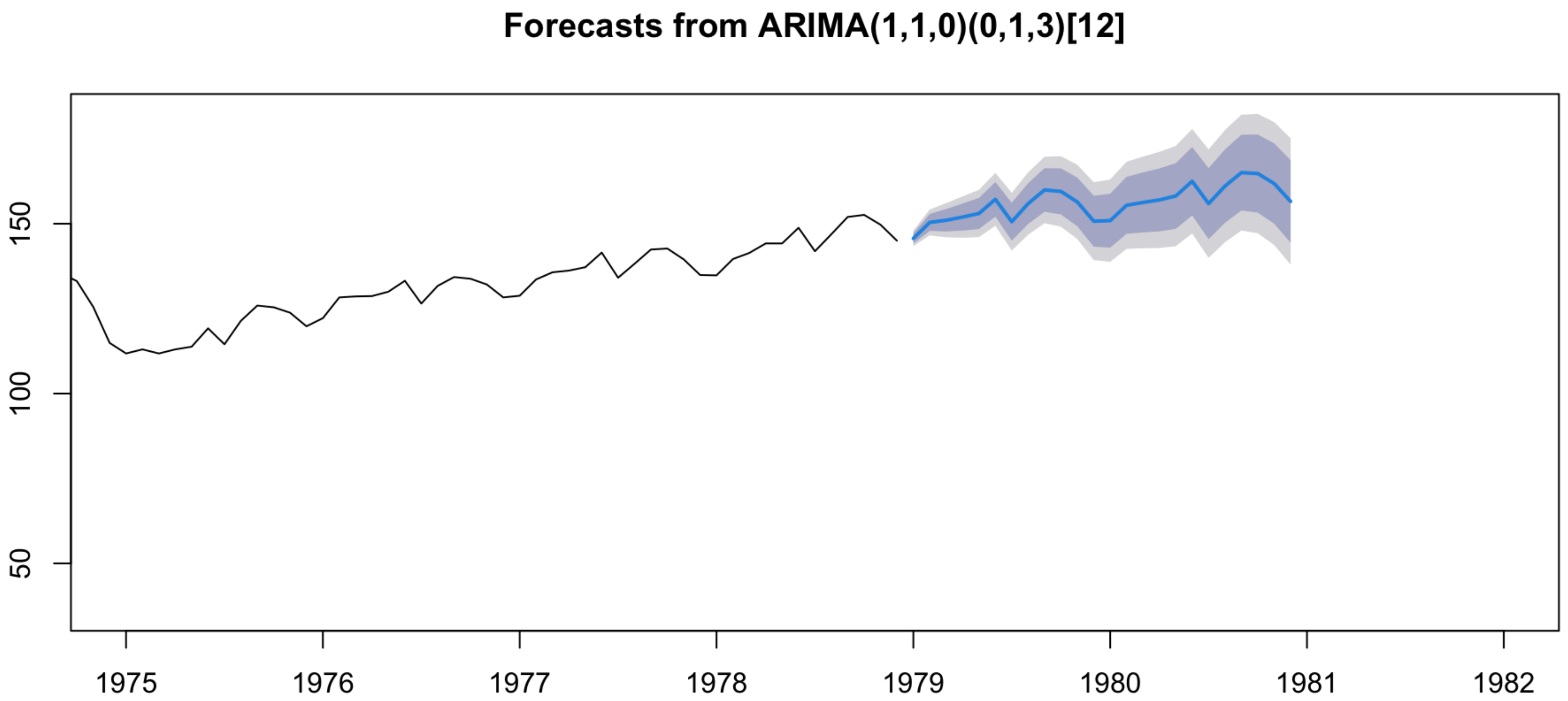
Model Forecast

```r
forecast::forecast(fr_m4.1) %>% plot()
```



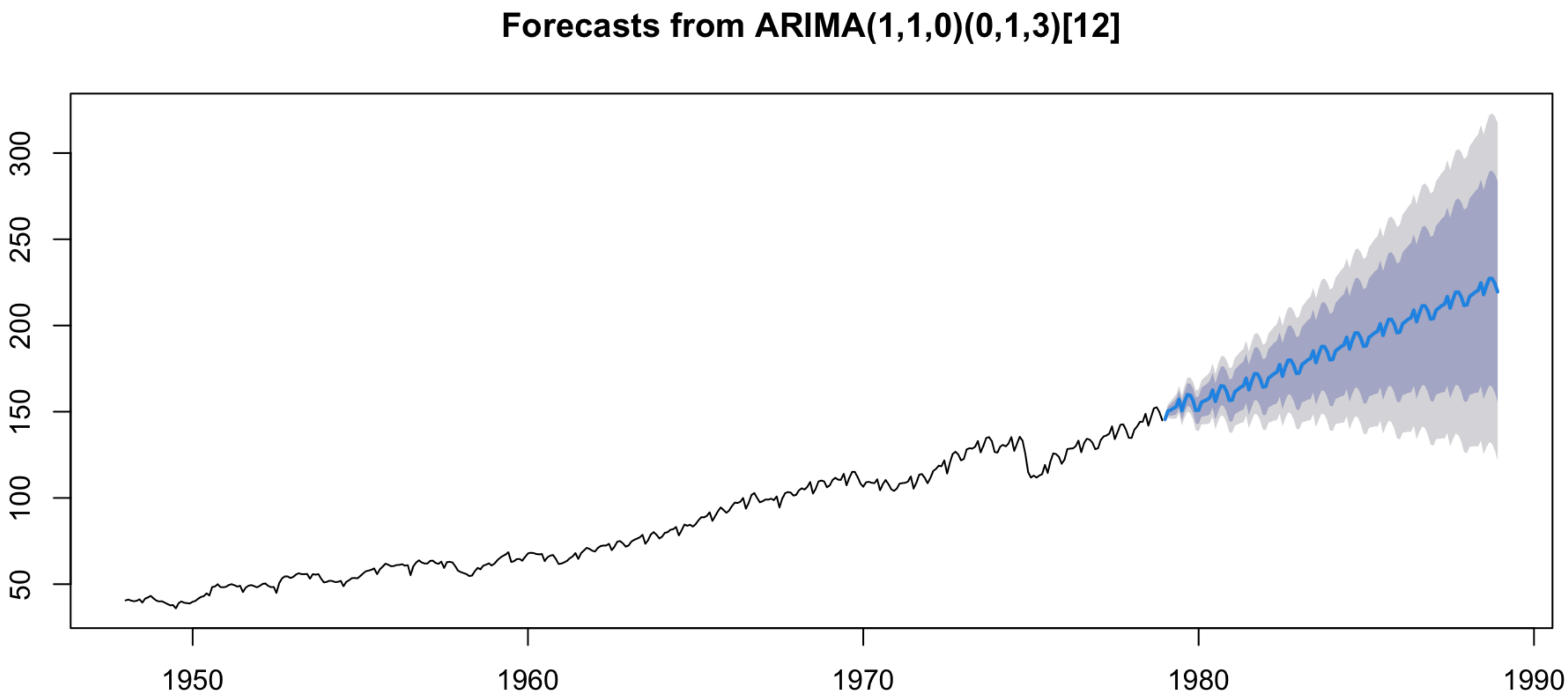
Model Forecast (cont.)

```r
forecast::forecast(fr_m4.1) %>% plot(xlim=c(1975,1982))
```



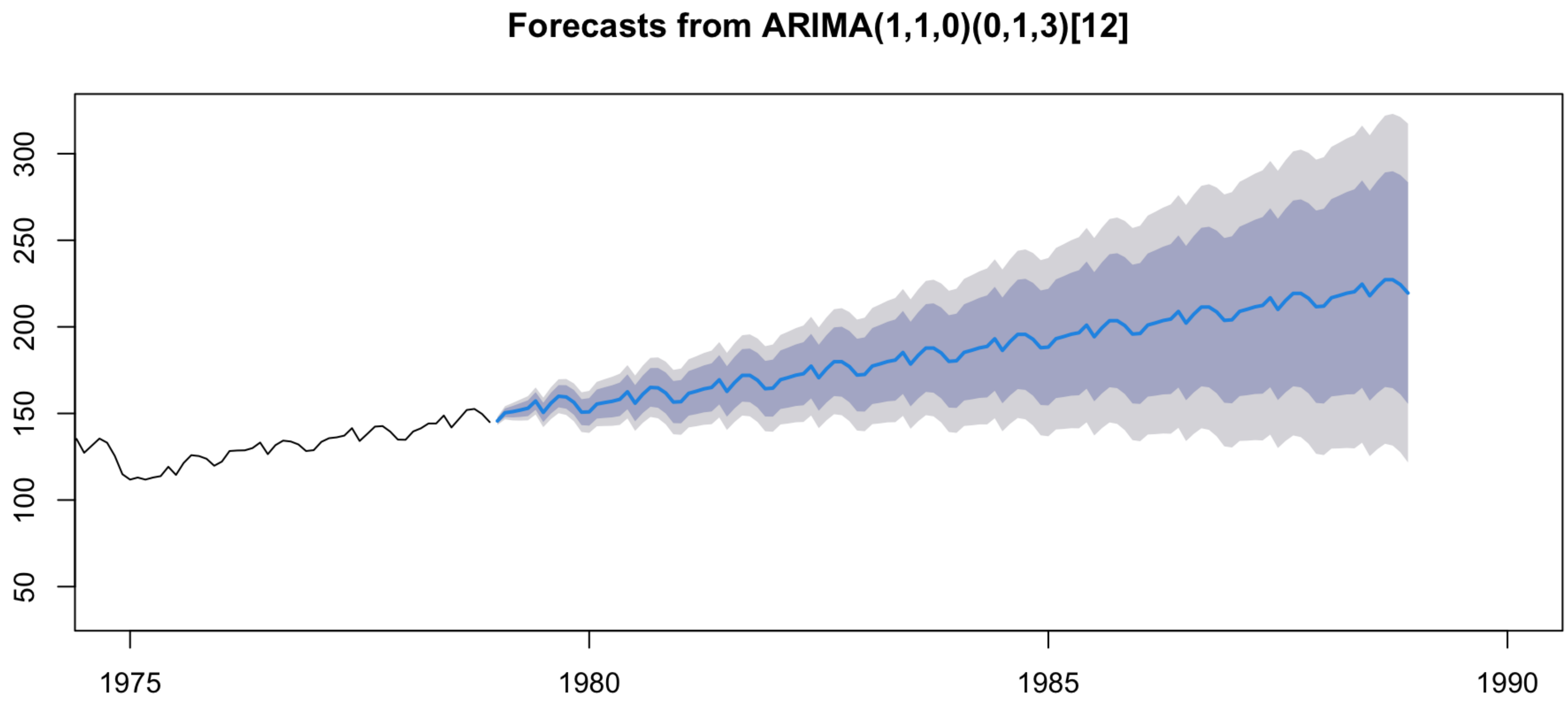
Model Forecast (cont.)

```
forecast::forecast(fr_m4.1, 120) %>% plot()
```

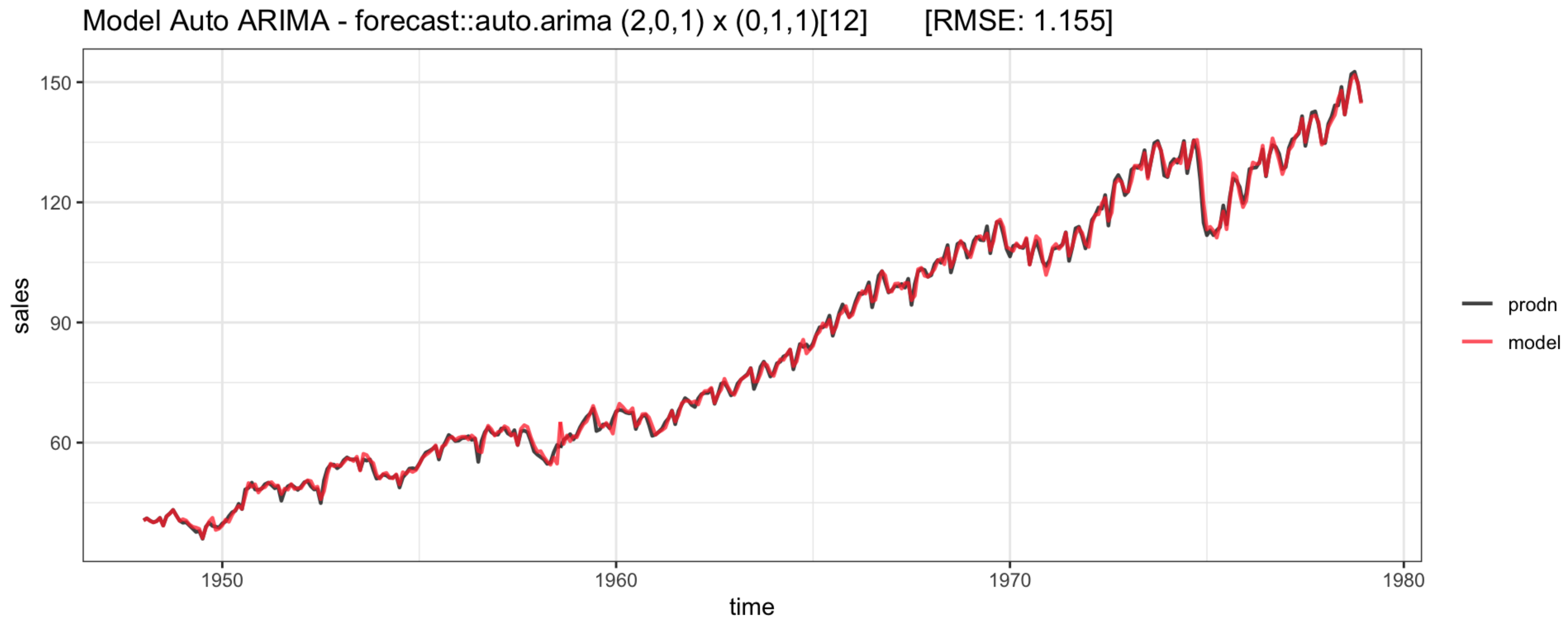


Model Forecast (cont.)

```
forecast::forecast(fr_m4.1, 120) %>% plot(xlim=c(1975,1990))
```



Auto ARIMA - Model Fit



Exercise - Cortecosteroid Drug Sales

Monthly cortecosteroid drug sales in Australia from 1992 to 2008.

```r
data(h02, package="fpp")
forecast::ggtsdisplay(h02, points=FALSE)
```